How to be (non-)specific

Maria Aloni Marco Degano

Abstract

Indefinites are known to give rise to different scopal (specific vs. non-specific) and epistemic (known vs. unknown) uses. Farkas and Brasoveanu [2020] explained these specificity distinctions in terms of stability vs. variability in value assignments of the variable introduced by the indefinite. Typological research [Haspelmath, 1997] showed that indefinites have different functional distributions with respect to these uses. In this work, we present a formal framework where Farkas and Brasoveanu [2020]'s ideas are rigorously formalized. We develop a two-sorted team semantics which integrates both scope and epistemic effects. We apply the framework to explain typological variety of indefinites, showing that only lexicalized indefinites have convex meanings in our system [Gardenfors, 2014, Steinert-Threlkeld et al., 2023]. We account for the restricted distribution and licensing conditions of different indefinites, and some diachronic developments of indefinite forms. We also focus on a particular class of indefinites, called epistemic indefinites [Alonso-Ovalle and Menéndez-Benito, 2017].

1 Introduction

Indefinites display a great variety in form and meaning across languages. This paper deals with two core phenomena in the domain of indefinite pronouns and determiners, and it examines them from a cross-linguistic viewpoint. First, specific and non-specific interpretations. Example (1) is an illustration:

(1) Ali wants to buy a mug.
   a. Specific: There is a specific mug which Ali wants to buy.
   b. Non-specific: Ali wants to buy a mug, any mug would do.

The ambiguity in (1) reflects also the scope behaviour of the indefinite with respect to the attitude verb *want*: *a mug* receives wide scope in (1a) and narrow-scope in (1b).[1]

Second, indefinites are known to give rise to different epistemic inferences with respect to the identity of the referent:[2]

[1][Fodor and Sag, 1982, Farkas, 1981, Reinhart, 1997, Kratzer, 1998, Winter, 1997, Schwarzschild, 2002, Brasoveanu and Farkas, 2011, Charlow, 2020].

[2][Fodor and Sag, 1982, Farkas, 1994, Kamp and Bende-Farkas, 2019].

(2) A linguist participated in the event.

a. Known: The speaker knows which linguist participated in the event.

b. Unknown: The speaker doesn't know which linguist participated in the event.

In a recent introductory article Farkas and Brasoveanu [2020] examined these distinctions between scopal and epistemic specificity.[3] They argued that these notions are related to stability versus variability of reference across different assignments of the variable introduced by the indefinite. Their work ended with two challenges. First, new theoretical tools need to be developed or refined to rigorously study these differences in stability and variability. Second, the relevant linguistic phenomena underlying these distinctions need to be carefully investigated.

For the first challenge, we develop a novel formal framework using tools from team semantics and dependence logic.[4] We show that our account captures both specific vs. non-specific and known vs. unknown uses. For the second challenge, languages mark these specificity and epistemic distinctions in the lexical meaning of particular indefinite forms. We will refer to such indefinites as *marked indefinites*. To make our discussion concrete, and typological comparisons possible, we rely on the work of Haspelmath [1997], who examined the functional distributions of indefinites in 40 languages. We show that our account captures the typological variety of marked indefinites within and across languages, explaining also why certain types of indefinites are unattested as a failure of convexity. We further account for the restricted distributions and licensing conditions of these indefinites. Our framework predicts also some diachronic developments of indefinites in terms of semantic weakening.

This paper is structured as follows. Section 2 outlines the core data of our investigation and how languages mark specificity distinctions cross-linguistically. Section 3 introduces our formal framework with the relevant technical machinery and background notions. Section 4 shows how this framework can be applied to model marked indefinites, together with several properties and phenomena associated with them. In particular, we focus on the typological variety of indefinites (Section 4.3), licensing restrictions (Section 4.4), the diachronic development of indefinites (Section 4.8) and some remarks on functional specificity (Section 4.9). Section 4.5 extends our framework with negation and examines the behavior of marked indefinites under negation. Section 4.6 extends our framework with modality and its interaction with indefinites. Section 4.7 is dedicated to epistemic indefinites, a well known class of indefinites in the semantic literature. Section 5 concludes.

[3]In their work, they also introduced the notion of partitive specificity, which we do not address here.

[4][Hodges, 1997, Väänänen, 2007a,b, Galliani, 2012, 2021].

2 Indefinites across languages

In Section 1, we examined different specificity and epistemic readings associated with indefinites. Example (3) illustrates these contrasts for English *someone*:

(3) a. Specific known (SK): Someone called. I know who.

b. Specific unknown (SU): Someone called. I do not know who.

c. Non-specific (NS): John needs to find someone for the job.

Cross-linguistically, languages developed lexicalized form with restricted distributions with respect to the uses in (3). For instance, German *irgend-* is incompatible with SK, as the infelicitous continuation in (4) shows:

(4) Irgendein Student hat angerufen. #Rat mal wer?
some student has called. guess who?
'Some (unknown) student called. #Guess who?'
(from Haspelmath [1997])

Another relevant example is Russian *-nibud'*, which is not allowed in episodic contexts and can only be interpreted non-specifically:

(5) *Ivan včera kupil kakuju-nibud' knigu.
Ivan yesterday bought which-INDEF. book.
'Ivan bought some [non-specific] book yesterday.'

Haspelmath [1997] examined indefinites' systems in 40 languages and developed a functional map of indefinites with nine main functions.[5] Figure 1 displays a semantic map for the German indefinite *irgend-*, where the colored area indicates the possible functions available for *irgend-*.

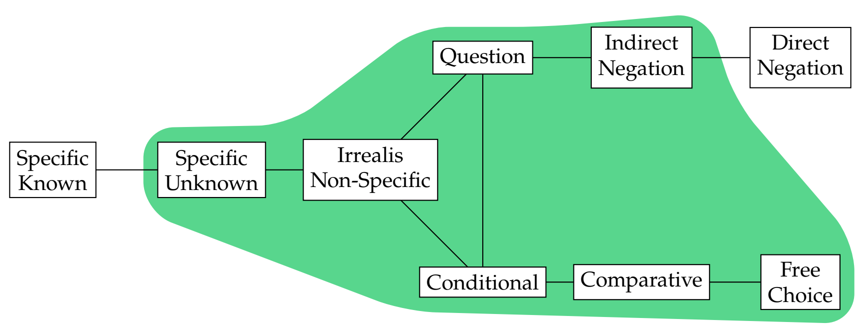


Figure 1: Haspelmath's map for German *irgend-*

[5] Haspelmath [1997] restricted his analysis to indefinite pronouns and determiners formed with indefinite markers (e.g., the English *some-* or *any-*) which occur in a series (e.g., *some-thing*, *some-where*, ...). This excludes from our work expressions such as *a certain*, which however have a specific-like flavour. An interesting research question is to determine if the behaviour of indefinites marked for specific uses parallels entirely specificity markers like *certain* in combination with indefinite articles.

TYPE	FUNCTIONS SK	SU	NS	EXAMPLE
(i) unmarked	✓	✓	✓	Italian *qualcuno*
(ii) specific	✓	✓	✗	Georgian *-ghats*
(iii) non-specific	✗	✗	✓	Russian *-nibud'*
(iv) epistemic	✗	✓	✓	German *irgend-*
(v) specific known	✓	✗	✗	Russian *koe-*
(vi) SK + NS	✓	✗	✓	unattested
(vii) specific unknown	✗	✓	✗	Kannada *-oo*

Table 1: Possible Types of Indefinites

As said, in this work we will mainly focus on the 3 functions specific known (SK), specific unknown (SU) and non-specific (NS) on the left part of the map.

Given the importance of the data considered by Haspelmath [1997] for our work, some words on the classification criteria used in his typological research are in order, especially for the specific vs. non-specific dimension. Indefinites with specific uses presuppose the existence of their referent (i.e., they can be paraphrased with a *there*-insertion construction). They can introduce discourse referents, as they allow continuations with appropriate pronominal expressions. On the other hand, non-specific indefinites are ungrammatical in episodic contexts and need a licensing operator, such as a modal or a *bona fide* quantifier. Note that under this analysis of specificity, specific indefinites admit only wide-scope readings, and thus the notion of specificity considered here amounts to scopal specificity. We will return to this issue in Section 4.9.

Combinations of these three functions lead to 7 possible indefinite types, summarized in Table 1 together with a relevant example.

Unmarked indefinites don't have any restriction with respect to these functions; specific indefinites admit only specific uses (SK and SU); non-specific indefinites admit only NS uses; and so-called epistemic indefinites allow for both SU and NS uses. The last two types deserve some remarks. Type (vi), encoding SK and NS but not SU, is unattested in the data collected by Haspelmath [1997]. Type (vii), admitting only SU uses, is very infrequent: out of the 40 languages that Haspelmath [1997] examined, only 1 has such indefinite, Kannada.[6]

Table 2 displays some within-language distinctions. Generalizations are difficult to make, given the limited amount of data.[7] Nevertheless, in the data collected by Haspelmath [1997], we see that overall the combination specific +

[6]A Dravidian language spoken mainly in Karnataka in south-western India. Kannada is a determinerless language and, as such, bare nouns are ambiguous between definite and indefinite uses. It might be possible that this facilitated the development of a specific form with unknown uses, since the definite already encodes familiarity with the referent. For more on the uses of Kannada bare nouns, see Srinivas and Rawlins [2021] and for the Kannada indefinite system in general, Bhat [2011].

[7]Moreover, we also note that there are equivalent expression (e.g., *a specific*) which, albeit not being indefinites, have meanings similar to some of the marked indefinites we consider here.

LANGUAGE	INDEFINITE	FUNCTIONS SK	SU	NS	TYPE
Italian	un qualche	✗	✓	✓	epistemic
	qualcuno	✓	✓	✓	unmarked
Russian	koe-	✓	✗	✗	specific known
	-to	✗	✓	✓	epistemic
	-nibud′	✗	✗	✓	non-specific
Japanese	-ka	✓	✓	✓	unmarked
Turkish	bir	✓	✓	✓	unmarked
	herhangi	✗	✓	✓	epistemic
German	etwas	✓	✓	✓	unmarked
	irgend	✗	✓	✓	epistemic
Georgian	-ghats	✓	✓	✗	specific
	-me	✗	✗	✓	non-specific
Ossetic	-dær	✓	✓	✗	specific
	is-	✗	✗	✓	non-specific
Kazakh	bir	✓	✓	✓	unmarked
	älde	✓	✓	✗	specific
Kannada	-oo	✗	✓	✗	specific unknown
	-aadaruu	✗	✗	✓	non-specific

Table 2: Marked indefinites across languages

non-specific is very common. And also the epistemic type is quite widespread. An important question, which will address in the coming sections, is to determine why such indefinites are so widespread. It is also quite typical that if a language has an indefinite marked only for specific uses, then indefinites of the epistemic kind are absent. In the case of Russian, we observe that there are two marked indefinites[8] to express NS: the epistemic *-to*, which also admits SU uses; and the non-specific *-nibud′*, which only admits non-specific uses. However, Russian speakers tend to select *-nibud′* for NS and *-to* for SU. Why then *-to* maintained its NS uses and did not become a specific unknown indefinite?

In the next section, we will develop a formal framework which will help us to address these questions, together with other several properties and puzzles associated with marked indefinites. In particular, we will account for the variety of marked indefinites in Table 1, including a principled explanation of the cross-linguistic absence of the SK + NS combination. (Section 4.3); the restricted distribution and licensing conditions of non-specific indefinites (Section 4.4); the interaction with negation (Section 4.5) and modality (Section 4.6); the meaning of epistemic indefinites (Section 4.7); the diachronic pathway from non-specific to epistemic (Section 4.8); and how marked indefinites interact with scope (Section 4.10).

[8]Russian has also other indefinites which might admit non-specific uses. We do not include them here, as they are commonly considered to be tied to different registers.

3 Two-sorted Team Semantics

Traditionally, formulas are interpreted with respect to a single evaluation point. In team semantics, formulas are interpreted with respect to sets of points, rather than single ones. These evaluations points can be valuations (as in propositional team logic, Yang and Väänänen [2017]), assignments (as in first-order team semantics, Galliani [2021], Lück [2020]) or possible worlds (as in team-based modal logic, Aloni [2022], Lück [2020]). This *set* of evaluations is usually called a *team*.

As a simple example, let us just consider a propositional case described in Table 3. The team $Z = \{i, j\}$ is composed of two valuations (i and j), assigning truth values to propositional atoms. As we will see, Z does not make p true, since p is not satisfied in all assignments of the team, but it makes q true, as q is satisfied in both i and j.

p	q	Z
1	1	i
0	1	j

Table 3: Simple Team over propositions

In what follows, we will work with a *two-sorted* first-order framework, with two sorts of entities, individuals in D and possible worlds in W, with variables ranging over each set. We define the language of our logical system as follows. In the rest of the section, we will clarify the underlying idea behind a two-sorted team semantics and the language defined below.

Definition 1 (Language) *Given a first-order signature σ (composed of individual constants $c \in C$, and predicates $P^n \in \mathcal{P}^n$ with $n \in \mathbb{N}$), and individual variables $z_d \in \mathcal{Z}_d$ and world variables $z_w \in \mathcal{Z}_w$, the terms and formulas of our language are defined as follows:*[9]

$$t ::= c | z_d | z_w$$

$$\phi ::= P(\vec{t}) \,|\, \neg P(\vec{t}) \,|\, t = t' \,|\, \phi \vee \psi \,|\phi \wedge \psi \,|\, \exists_{strict} z \phi \,|\, \exists_{lax} z \phi \,|\forall z \phi| \, dep(\vec{z}, z) \,|\, var(\vec{z}, z)$$

Definition 2 (Two-sorted model) *A two-sorted model is a triple $M = \langle D, W, I \rangle$ composed of a domain of individuals $Dom_d(M) = D$, a domain of worlds $Dom_w(M) = W$, and an interpretation function I assigning an element of D to every individual constant symbol and sets of n-tuples constructed from W and D to every n-ary predicate symbol.*

A two-sorted first-order team is just a set of assignments mapping world variables to elements of W and individual variables to elements of D. We first define a variable assignment and then a team.[10]

[9] *$\vec{t}$ stands for an arbitrary sequence $t_1, \ldots, t_n$.*

[10] To keep the definitions general, we indicate the sort in the subscript. z_d and z_w will be individual and world variables respectively. Similarly, e_d will be an element of D and e_w an element of W. Later, we will use more conventional labels.

Definition 3 (Variable Assignments) *Given a two-sorted first-order model* $M = \langle D, W, I\rangle$ *and a set of variables* $Z = Z_d \cup Z_w$*, an assignment* i *is a function from* Z *to* $D \cup W$*, s.t.* $i(z) \in D$ *if* $z \in Z_d$ *and* $i(z) \in W$ *if* $z \in Z_w$*. For any variable* z_* *and any element* e_* *with* $* \in \{d, w\}$*, we write* $i[e_*/z_*]$ *for the assignment function with domain* $Z \cup \{z_*\}$ *s.t. for all variable symbols* $l \in Z \cup \{z_*\}$*:*

$$i[e_*/z_*](l) = \begin{cases} e_* & \textit{if } l = z_* \\ i(l) & \textit{otherwise} \end{cases}$$

For every assignment i*, every sequence* $\vec{e} = e_1, \ldots, e_n$ *and* $\vec{z} = z_1, \ldots, z_n$*, we write* $i[\vec{e}/\vec{z}]$ *as an abbreviation for* $i[e_1/z_1] \ldots [e_n/z_n]$*.*

A team in our framework is, as anticipated, a set of variable assignments:

Definition 4 (Team) *Given a two-sorted first-order model* $M = \langle D, W, I\rangle$ *and a set of variables* $Z = Z_d \cup Z_w$*, a team* T *over* M *with domain* $Dom(T) = Z$ *is a set of assignments* i *with domain* Z*.*

T	v	x
i_1	v_1	d_1
i_2	v_2	d_2

Table 4: Example of a two-sorted first order team $T = \{i_1, i_2\}$ with domain $Z = \{v, x\}$, and $D = \{d_1, d_2, \ldots\}$, $W = \{v_1, v_2, \ldots\}$.

3.1 Teams as information states

Teams represent information states of speakers. In initial teams only factual information is represented, encoded by a designated variable $v \in Z_w$.

Definition 5 (Initial Team) *A team* T *is initial iff* $Dom(T) = \{v\}$*.*

The possible values of v in a team represent different ways the world might be (epistemic possibilities). Intuitively, a team where v receives only one value is of maximal information.

Definition 6 (Team of Maximal Information) *A team* T *is of maximal information iff* $i(v) = j(v)$ *for all* $i, j \in T$*.*

In initial teams, only factual information is present. Then, operations of assignment extension add discourse information to the team. This leads to defining a sentence as felicitous if there is an initial team which supports it:

Definition 7 (Felicitous sentence) *A sentence is felicitous/grammatical if there is an initial team which supports it.*

In the team represented in Table 5, the first row indicates the variables present in the team and the rows below the values assigned by the assignments in the team. The first column in yellow encodes factual information and conveys that the epistemic possibilities the speaker entertains are v_1, v_2 and up to v_n. Discourse information is then added by operations of assignment extensions, which can introduce individual or world variables. As said, teams encode the information state of the speaker. For instance, in Table 5 the speaker is certain about - or *knows* - the value of x, since x is constant across all her epistemic possibilities. However, the speaker does not know the value of y. World variables, like w, are introduced to model modals or attitudes verbs, as we will see in the next sections.

v	x	w	y	...
v_1	a	w_1	b_1	...
v_2	a	w_2	b_2	...
...	a	...	...	...
v_n	a	w_n	b_n	...

Table 5: Team as information state (initial team in yellow)

3.2 Assignment extensions

Our assignment extensions are based on similar operations in dynamic and team semantics [Groenendijk and Stokhof, 1991, Dekker, 1993, Aloni, 2001, Väänänen, 2007b, Galliani, 2012]:

Definition 8 (Universal Extension) *Given a model $M = \langle D, W, I\rangle$, a team T and a variable z_* with $* \in \{d, w\}$, the universal extension of T with z_*, $T[z_*]$ is defined as follows:*

$$T[z_*] = \{i[e_*/z_*] : i \in T \textit{ and } e_* \in Dom_*(M)\}$$

Universal extensions consider all assignments that differ from the ones in T only with respect to the value of z_*. Table 6(b) is an example, assuming the initial team in Table 6(a) and a domain of two individuals. Note that universal extensions are unique.

Definition 9 (Strict Functional Extension) *Given a model $M = \langle D, W, I\rangle$, a team T and a variable z_* with $* \in \{d, w\}$, the strict functional extension of T with z_*, $T[f_s/z_*]$ is defined as follows:*

$$T[f_s/z_*] = \{i[f_s(i)/z_*] : i \in T\}, \textit{for some strict function } f_s : T \to Dom_*(M)$$

Strict functional extensions assign only one value to the variable for each assignment in the original team T. Table 6(c) shows one of the four possible examples, assuming the initial team in Table 6(a) and a domain of two individuals.

Definition 10 (Lax Functional Extension) *Given a model $M = \langle D, W, I \rangle$, a team T and a variable z_* with $* \in \{d, w\}$, the lax functional extension of T with z_*, $T[f_l/z_*]$ is defined as follows:*

$$T[f_l/z_*] = \{i[e_*/z_*] : i \in T \text{ and } e_* \in f_l(i)\}, \text{for some lax function } f_l : T \longrightarrow \wp(Dom_*(M)) \backslash \{\varnothing\}$$

Lax functional extensions amount to assign one or more values to the variable for each original assignment in T. Table 6(d) shows one of the nine possible examples, assuming the initial team in Table 6(a) and a domain of two individuals.

(a)

v	T
v_1	i_1
v_2	i_2

(b)

v	y	$T[y]$
v_1	$\rightarrow d_1$	i_{11}
	$\rightarrow d_2$	i_{12}
v_2	$\rightarrow d_1$	i_{21}
	$\rightarrow d_2$	i_{22}

(c)

v	y	$T[f_s/y]$
v_1	$\longrightarrow d_1$	i_{11}
v_2	$\longrightarrow d_2$	i_{22}

(d)

v	y	$T[f_l/y]$
v_1	$\longrightarrow d_2$	i_{12}
v_2	$\rightarrow d_1$	i_{21}
	$\rightarrow d_2$	i_{22}

Table 6: Initial Team (a), universal y-extension (b), strict functional y-extension (c), and lax functional y-extension (d), with $D = \{d_1, d_2\}$

3.3 Dependence and Variation atoms

Team semantics frameworks are often equipped with dependence atoms - expressions which impose conditions of dependence on the variables' values given by the different assignments.[Väänänen, 2007a, Galliani, 2021]. In this work, we adopt the following two atoms:

Definition 11 (Dependence Atom)
$M, T \models dep(\vec{x}, y) \Leftrightarrow$ *for all* $i, j \in T : i(\vec{x}) = j(\vec{x}) \Rightarrow i(y) = j(y)$

Definition 12 (Variation Atom)
$M, T \models var(\vec{x}, y) \Leftrightarrow$ *there is* $i, j \in T : i(\vec{x}) = j(\vec{x})$ & $i(y) \neq j(y)$

The first atom in Definition 11 says that if any two assignments agree on the value of $\vec{x}$, they also agree on the value of y (i.e. the value of y is *dependent* on the value of $\vec{x}$). The variation atom in Definition 12 corresponds to the metalinguistic negation of the definition of Dependence Atom above, and as such it encodes the failure of functional dependence.[11] It is valid when there is at least a pair of assignments for which the value of y varies and $\vec{x}$ is the same. Table 7 displays a team of three assignments together with some illustrations.

T	x	y	z	l
i	a_1	b_1	c_1	d_1
j	a_1	b_1	c_2	d_1
k	a_3	b_2	c_3	d_1

$dep(x,y)$ ✓	$var(x,z)$ ✓
$dep(\emptyset,l)$ ✓	$var(\emptyset,x)$ ✓
$dep(xy,z)$ ✗	$var(x,y)$ ✗

Table 7: Dependence and Variation atoms - Illustrations

In Table 7, we have that $dep(x,y)$, since for any assignment i, j and k, the value of x determines the value of y. But we do not have $dep(xy,z)$ (consider for example i and j: $i(xy) = j(xy)$, but $i(z) \neq j(z)$). It also holds that $var(x,z)$ since $i(x) = j(x)$ but $i(z) \neq j(z)$. A special case are constancy atoms of the form $dep(\emptyset,l)$, which is valid when l receives the same value across all assignments; and variation atoms of the form $var(\emptyset,y)$, which is valid when y receives different values across at least a pair of assignments.

We now give precise rules for semantic clauses of the formulas of our language [Hodges, 1997, Väänänen, 2007a, Galliani, 2012].[12]

Definition 14 (Semantic Clauses)

$$
\begin{array}{lcl}
M,T \models P(t_1,\ldots,t_n) & \Leftrightarrow & \forall j \in T: \langle j(t_1),\ldots,j(t_n)\rangle \in I(P^n) \\
M,T \models \neg P(t_1,\ldots,t_n) & \Leftrightarrow & \forall j \in T: \langle j(t_1),\ldots,j(t_n)\rangle \notin I(P^n) \\
M,T \models t_1 = t_2 & \Leftrightarrow & \forall j \in T: j(t_1) = j(t_2) \\
M,T \models \neg(t_1 = t_2) & \Leftrightarrow & \forall j \in T: j(t_1) \neq j(t_2) \\
M,T \models \phi \wedge \psi & \Leftrightarrow & M,T \models \phi \text{ and } M,T \models \psi \\
M,T \models \phi \vee \psi & \Leftrightarrow & T = T_1 \cup T_2 \text{ for teams } T_1 \text{ and } T_2 \text{ s.t. } M,T_1 \models \phi \\
 & & \text{and } M,T_2 \models \psi \\
M,T \models \forall z\, \phi & \Leftrightarrow & M,T[z] \models \phi \\
M,T \models \exists_{\text{strict}} z\, \phi & \Leftrightarrow & \text{there is a strict function } f_s \text{ s.t. } M,T[f_s/z] \models \phi \\
M,T \models \exists_{\text{lax}} z\, \phi & \Leftrightarrow & \text{there is a lax function } f_l \text{ s.t. } M,T[f_l/z] \models \phi \\
M,T \models dep(\vec{x},y) & \Leftrightarrow & \text{for all } i,j \in T : i(\vec{x}) = j(\vec{x}) \Rightarrow i(y) = j(y) \\
M,T \models var(\vec{x},y) & \Leftrightarrow & \text{there is } i,j \in T : i(\vec{x}) = j(\vec{x})\ \&\ i(y) \neq j(y)
\end{array}
$$

[11]Our variation atom was briefly mentioned in Galliani [2012]. In dependence logics, a stronger version of the variation atom is typically considered:

Definition 13 (Variation Atom (Stronger Version))
$M,T \models VAR(\vec{x},y) \Leftrightarrow$ *for all* $i \in T$ *there is* $j \in T : i(\vec{x}) = j(\vec{x})\ \&\ i(y) \neq j(y)$

Note in fact that $VAR(\vec{x},y$, unlike $var(\vec{x},y)$, is downward closed like $dep(\vec{x},x)$, a property which typically simplifies the study of the underlying logics. Recently, Väänänen [2022] employed the stronger variation atom, called *anonymity atom* in his work, to model the notion of anonymity in database theory. See also Yang [2022] for some metatheoretical results on the propositional fragment of these logics.

[12]We will later introduce an intensional notion of negation. For negation in Dependence Logic, see Kontinen and Väänänen [2011].

Definition 15 (Entailment) *A formula ϕ entails a formula ψ, in symbols $\phi \models \psi$, if for all M and all T such that $M, T \models \phi$, we have $M, T \models \psi$.*

A first order literal is true in a team T iff it is true in all assignments in T. We allow negation only on first-order atoms and we assume that formulas are always in negation normal formal. We will return to negation in Section 4.5. A team T satisfies a conjunction $\phi \wedge \psi$ iff T satisfies ϕ and satisfies ψ. A team T satisfies a disjunction $\phi \vee \psi$ iff T is the union of two subteams, each satisfying one of the disjuncts.[13] We use the universal extension for the universal quantifier, and the strict and lax functional extensions for the strict and lax existentials.

It is interesting to observe that, except for the variation atom, all formulas in our language are downward closed ($T \models \phi$ and $T' \subseteq T$ imply $T' \models \phi$). The variation atom, instead, is upward closed ($T \models \phi$ and $T \subseteq T'$ imply $T \models \phi$), and therefore also union-closed ($T \models \phi$ and $T' \models \phi$ imply $T \cup T' \models \phi$). We note that for downward closed formulas, the strict and lax existentials are equivalent. In the next sections, we will see that the variation atom and its interaction with the two existentials will play an important role.

4 Applications

4.1 Indefinites as existentials and scope behaviour

We model indefinites as strict existentials ($\exists_{(s)trict)} x\ \phi$) and we interpret them *in-situ*.[14] With this very minimal assumption, and with the help of dependence atoms, we can already capture the scopal behavior typically associated with indefinites [Fodor and Sag, 1982, Reinhart, 1997, Kratzer, 1998]. Dependence atoms allow us to easily capture the different scope readings by specifying how the indefinite's variable co-varies with other operators. For instance, a sentence like (6) is ambiguous between three different readings, depending on the scope of *a doctor* with respect to the universal quantifiers.[15]

As base case, we assume a team of maximal information (i.e. the value of v is fixed). As shown in Table 8, $dep(v, y)$ yields a wide scope interpretation where the value of y is constant; $dep(vx, y)$ yields the intermediate reading where the value of y depends only on the first universal quantifier; and $dep(vxz, y)$ yields narrow scope where the value of y depends on both universal quantifiers.

[13] We are employing the so-called split or tensor disjunction [Väänänen, 2007b].

[14] Modelling indefinites as objects which map to the domain of our model is quite standard in frameworks working with a set of evaluation points, as in dynamic semantics. Moreover, we would like to mention Champollion et al. [2017], a recent relevant work which integrates dependence logics and dynamic plural logic. Champollion et al. [2017] adopts a variant of our strict existential together with a rigidity requirement comparable to our $dep(\varnothing, x)$ to model indefinites with a specific use. We thank Lucas Champollion for pointing out to us this interesting convergence.

[15] We give some concrete instantiation of the three readings. In the wide scope reading in (6a), there is a particular doctor (say Dr. Malcom), such that every kid ate every food that Dr. Malcom recommended. In the intermediate scope reading, for every kid, there is a doctor, say the paediatrician of each kid, such that all kids ate every food that their doctor recommended. In the narrow scope reading, the sentence is true also in cases of total co-variation between the doctors and the foods.

v	x	z	y
v_1	...	...	b_1
v_1	...	...	b_1
v_1	...	...	b_1
v_1	...	...	b_1

WS: $dep(v, y)$

v	x	z	y
v_1	a_1	...	b_1
v_1	a_1	...	b_1
v_1	a_2	...	b_2
v_1	a_2	...	b_2

IS: $dep(vx, y)$

v	x	z	y
v_1	a_1	c_1	b_1
v_1	a_1	c_2	b_2
v_1	a_2	c_3	b_3
v_1	a_2	c_4	b_4

NS: $dep(vxz, y)$

Table 8: Indefinites & Scope

(6) Every kid$_x$ ate every food$_z$ that a doctor$_y$ recommended.
   a. Wide scope $[\exists y/\forall x/\forall z]$: $\forall x \forall z \exists_s y(\phi \wedge dep(v, y))$
   b. Intermediate scope $[\forall x/\exists y/\forall z]$: $\forall x \forall z \exists_s y(\phi \wedge dep(vx, y))$
   c. Narrow scope $[\forall x/\forall z/\exists y]$: $\forall x \forall z \exists_s y(\phi \wedge dep(vxz, y))$

For what concerns scope, our approach is conceptually similar to Brasoveanu and Farkas [2011] and leads to the generalization in (7). In our framework, dependency relations are not part of the meaning of the existential, but they are evaluated as separate clauses. This allows us to work with a uniform entry for existentials and with a better behaved logical system.[16]

(7) Indefinites & Scope
An unmarked/plain indefinite $\exists_s x$ in syntactic scope of $O_{\vec{z}}$ allows all $dep(\vec{y}, x)$, with $\vec{y}$ included in $v\vec{z}$:

$$O_{z_1} \dots O_{z_n} \exists_s x(\phi \wedge dep(\vec{y}, x))$$

4.2 Specific Known, Specific Unknown and Non-Specific

In the example considered in the previous section in Table 8, we worked with a team of maximal information (i.e., the value of v was fixed). To model epistemic distinctions, we need to distinguish between full specificity (specific known) and what we called specific unknown: a specific individual, but epistemically not determined. We can capture the difference using possible worlds representing epistemic possibilities. In the former case, the specific individual will be constant across all epistemically possible worlds, while in the latter it will vary. The conditions in Table 9 make our strategy more precise.

Constancy means that the variable x is mapped to the same individual in every assignment, while *variation* guarantees that there is at least a pair of assignments in which x receives different values. Their v-counterparts relativize these notions to the designated world variable v: v-constancy means that the

[16]The generalization in (7) overgenerates. Unavailable readings can be ruled following a strategy similar to Brasoveanu and Farkas [2011]. We do not discuss this any further, as our main concerns here are the typological variety of indefinites and the integration of epistemic readings.

		v	x			v	x
constancy	$dep(\varnothing, x)$	$\ldots$	d_1	v-constancy	$dep(v, x)$	w_1	d_1
		$\ldots$	d_1			w_2	d_2
		v	x			v	x
variation	$var(\varnothing, x)$	$\ldots$	d_1	v-variation	$var(v, x)$	w_1	d_1
		$\ldots$	d_2			w_1	d_2

Table 9: Constancy and variation conditions

TYPE	FUNCTIONS			REQUIREMENT	EXAMPLE
	SK	SU	NS		
(i) unmarked	✓	✓	✓	none	Italian *qualcuno*
(ii) specific	✓	✓	✗	$dep(v, x)$	Georgian *-ghats*
(iii) non-specific	✗	✗	✓	$var(v, x)$	Russian *-nibud'*
(iv) epistemic	✗	✓	✓	$var(\varnothing, x)$	German *irgend-*
(v) specific known	✓	✗	✗	$dep(\varnothing, x)$	Russian *koe-*
(vi) SK + NS	✓	✗	✓	$dep(\varnothing, x) \veebar var(v, x)$	unattested
(vii) specific unknown	✗	✓	✗	$dep(v, x) \wedge var(\varnothing, x)$	Kannada *-oo*

Table 10: Marked Indefinites

value of x is constant given an epistemic possibility, whereas v-variation guarantees that there is at least an epistemic possibility in which x receives different values. With these conditions, we can logically characterize the specific known, specific unknown and non-specific functions (see 8). SK is captured by constancy, ensuring speaker knowledge; SU is captured by v-constancy, ensuring specificity, and variation, ensuring unknownness; NS is captured by v-variation, which as we will see will ensure scopal non-specificity.

(8) a. SK: $\exists x\, (\varphi(x, v) \wedge dep(\varnothing, x))$ [constancy]

b. SU: $\exists x\, (\varphi(x, v) \wedge dep(v, x) \wedge var(\varnothing, x))$ [v-constancy + variation]

c. NS: $\exists x\, (\varphi(x, v) \wedge var(v, x))$ [v-variation]

4.3 Variety

We have now all the ingredients to capture the variety of marked indefinites discussed in Section 2. As anticipated, we claim that marked indefinites come with particular restrictions with respect to the dependence and variation conditions examined in the previous section. We summarize our proposal in Table 10.

Unmarked indefinites, like English *someone*, don't have particular requirements, and they can in principle express all the functions that we considered. Specific indefinites are associated with 'v-constancy': the referent of the indefinite is the same in a given world, but it can possibly vary between worlds. The opposite condition, 'v-variation', forms the class of non-specific indefinites. Epistemic indefinites require 'variation': the referent of the indefinite must vary, possibly within the same world. 'Constancy' leads to specific known: a unique individual across all worlds.

Let us now turn to the last two types of Table 10, which require a more detailed explanation. The type 'specific known + non-specific' cannot be subsumed under a single atom. It requires that the referent satisfies either 'constancy' or 'v-variation', which are incompatible with each other.[17] Therefore, this type can only be captured by a (Boolean) disjunction of atoms, which explains the difficulty of finding a lexicalized indefinite encoding almost opposite meanings.[18] To our knowledge, there is no language which encodes this meaning in a particular form.

Moreover, type (vi) constitutes a clear violation of convexity, normally assumed as a constraint of lexicalizations [Steinert-Threlkeld et al., 2023, Enguehard and Chemla, 2021, Gardenfors, 2014]. For instance, all modified numerals are convex and there are no expressions which lexicalize meanings like 'more than five or less than two'. The underlying assumption is that numeral modifiers are defined upon a set of numbers which is linearly ordered, and no gaps are possible. In the same way, we claim that the meaning space which defines marked indefinites are the dependence and non-dependence conditions discussed in the present work. Figure 3 orders our atoms according to the degree of variation (from constancy to v-variation), and shows in which sense type (vi) creates a gap in the meaning space of marked indefinites.[19]

This point can be made more precise using the notion of a convex set of teams which formalises convexity in our framework:

Definition 16 (Convexity) *A set of teams P is convex iff for all T, T', T'' such that $T \subseteq T' \subseteq T''$, if $T \in P$ and $T'' \in P$, then $T' \in P$.*

It is easy to show that the Boolean union of the formulas associated with the SK and NS cells in our map, as in (9), define a property which does not satisfy convexity. A counterexample is given in Figure 2.

(9) SK + NS: $dep(\varnothing, x) \mathbin{\vee\!\!\!\vee} var(v, x)$

But this is crucially not the case for the other two possible combinations, which do define convex sets of teams, the former because $dep(v, x)$ is downward-closed, and the latter because $var(\varnothing, x)$ is upward-closed:

(10) SK + SU: $dep(\varnothing, x) \mathbin{\vee\!\!\!\vee} (var(\varnothing, x) \wedge dep(v, x)) \equiv dep(v, x)$

(11) SU + NS: $(var(\varnothing, x) \wedge dep(v, x)) \mathbin{\vee\!\!\!\vee} var(v, x) \equiv var(\varnothing, x)$

[17] Note in fact that $dep(\varnothing, x)$ implies $dep(v, x)$, which contradicts $var(v, x)$.

[18] To express such combination of functions, we would need a Boolean notion of disjunction: $M, T \models \phi \mathbin{\vee\!\!\!\vee} \psi \Leftrightarrow M, T \models \phi$ or $M, T \models \psi$. Note that $\mathbin{\vee\!\!\!\vee}$ is definable in our system:

$$\phi \mathbin{\vee\!\!\!\vee} \psi \equiv \exists x \exists y (dep(\varnothing, x) \wedge dep(\varnothing, y) \wedge (x = y \wedge \phi) \vee (x \neq y \wedge \psi))$$

[19] We observe that the conditions in Table 9 can be considered the most basic representation of constancy and variation requirements in the variables' assignment values, and in this sense they constitute minimal meaning elements of the meaning space of indefinites.

v	x
v_1	d_1

(a) T

v	x
v_1	d_1
v_2	d_2

(b) T'

v	x
v_1	d_1
v_1	d_2
v_2	d_2

(c) T''

Figure 2: Failure of Convexity for SK + NS. In the teams above, it holds that $T \subseteq T' \subseteq T''$. Moreover, $T \models dep(\varnothing, x) \mathbin{\vee\!\!\!/} var(v, x)$, since $T'' \models dep(\varnothing, x)$. $T'' \models dep(\varnothing, x) \mathbin{\vee\!\!\!/} var(v, x)$, since $T \models var(v, x)$. But $T' \not\models dep(\varnothing, x)) \mathbin{\vee\!\!\!/} var(v, x)$.

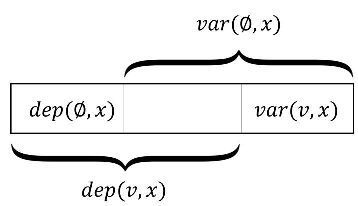


Figure 3: Meaning Space of Marked Indefinites

This gives us a principled explanation of the specific ordering among functions assumed in the original Haspelmath's map, namely SK-SU-NS. A natural constraint on implicational maps is that properties expressed by contiguous cells must satisfy convexity. If we had ordered the functions differently than assumed in the original map, e.g., SK-NS-SU, or SU-SK-NS, this constraint would not have been satisfied.

The last type, specific unknown, requires two atoms: 'v-constancy' for specificity and 'variation' for unknown. Crucially, only one language among the ones examined by Haspelmath [1997] had such indefinite. We claim that complexity is the reason. Specific unknown requires two atoms, and a possible lexicalization is therefore less likely to occur.

This analysis also allows us to answer the question at the end of Section 1. Russian has a dedicated indefinite for NS uses (*-nibud'*) and also an epistemic indefinite (*-to*) which express both NS and SU. In practice, speakers select almost always *-to* for SU and *-nibud'* for NS. The preferential use of SU for *-to* has arguably a pragmatic root: speakers are aware that there is an alternative form with only NS uses. But still Russian maintains *-to* as an epistemic, since turning *-to* into a specific unknown would make it more complex in the sense delineated here. An interesting balance between the language user and the language system.

We would like to conclude this section with an interesting parallelism between our dependence and variation conditions and Aristoteles' Square of Opposition. Figure 4 displays the traditional Aristoteles' Square of Opposition, which is a collection of logical relations between four main categorical propositions.[20] The corners are traditionally considered to be propositions, but Figure 4 displays the corresponding determiner (e.g., *Every A is B* for *Every*). Typically, only three corners of the square correspond to simple lexical items across languages. For instance, English lexicalized *every*, *some*, and *no*, but not *not every* as a simple determiner.[21]

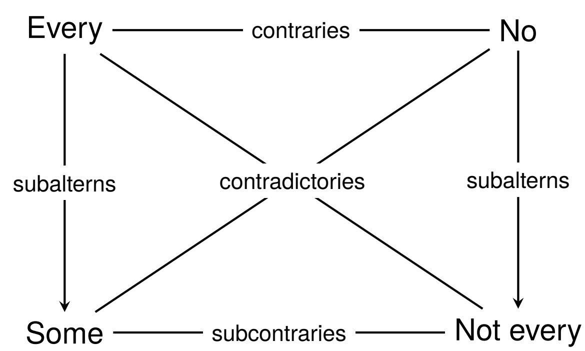


Figure 4: Aristotle's Square of Opposition

Interestingly, our dependence conditions along the dimensions of (v-)constancy and (v)-variation give rise to the same logical relationships observable in the standard Aristotelian square. Figure 7 displays our 'Dependence Square of Opposition'. Crucially, each corner corresponds to one of the lexicalized marked indefinites discussed in the previous section.

In the traditional Aristotelian Square, each corner corresponds to the four basic ways in which categorical propositions can be formed. Similarly, the Dependence Square of Opposition corresponds to the four basic ways in which marked indefinites can be formed. Moreover, we note the absence of the indefinite 'SK + NS' and 'specific unknown', reinforcing the idea the indefinites present in the Square are simpler and more frequent, while the others are unattested or rare.[22]

[20]We remind the reader of the classical terminology:

- Contraries: Two propositions are *contraries* iff they can be both false, but not both true.
- Contradictories: Two propositions are *contradictories* they cannot be both true and they cannot be both false.
- Subcontraries: Two propositions are *subcontraries* iff they cannot both be false but can both be true.
- Subalternation: A proposition A subalternates a proposition B iff A implies B.

Note that the relationships in Figure 4 holds assuming that *Every* and *No* have existential import, while *Some* and *Not Every* do not.

[21]A similar pattern can be observed in the domain of temporal adverbs. English lexicalizes *always*, *never* and *sometimes*, but no corresponding adverb in the lower right corner of the square.

[22]An open question is why the lower right corner of the ' Dependence Square of Opposition' is lexicalized, unlike the cases we mentioned before for determiners or adverbs.

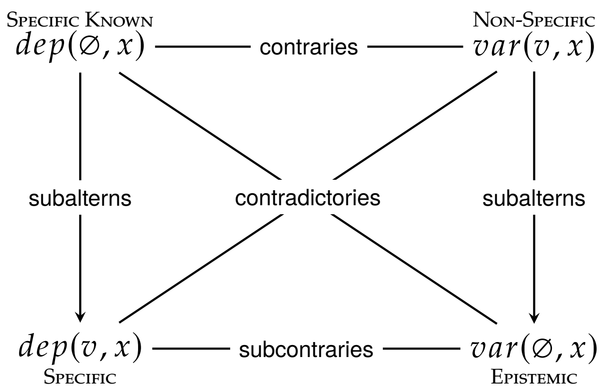


Figure 5: Dependence Square of Opposition

4.4 Non-specific Indefinites: Licensing & Variation

Non-specific indefinites are quite wide-spread cross linguistically[23] and they have received some consideration in the literature. Often, non-specific indefinites go under the name of 'dependent' indefinites from Farkas [1997]. We note, in passing, that non-specific indefinites typically allow for a wider range of licensors, whereas so-called dependent indefinites are typically not licensed by modals. We leave a detailed comparision between the two notions for future work, as a thorough analysis would involve a serious treatment of distributivity and plurality.

Non-specific indefinites cannot occur freely in episodic sentences, but they need to be licensed by an operator (a universal quantifier, a modal, an attitude verb, . . .). Examples (12) and (13) illustrate the case of Russian *-nibud'*.

(12) *Ivan včera kupil kakuju-nibud' knigu.
Ivan yesterday bought which-INDEF. book.
'Ivan bought some [non-specific] book yesterday.'

(13) Ivan hotel spet' kakuju-nibud' pesniu.
Ivan want-PAST sing-INF which-INDEF. song.
'Ivan wanted to sing some [non-specific] song.'

In Section 3.1, we have defined what counts as an initial team and the conditions under which a sentence is grammatical. This, together with the $var(v, x)$ requirement for non-specific indefinites, is enough to explain cases like (12) and (13).

To see this, suppose that we have an initial team where v is assigned to two worlds (see (a) in Table 11). Recall that non-specific indefinites trigger the v-variation condition: $\exists_s x(\phi(x, v) \wedge var(v, x))$. In order to satisfy $var(v, x)$, there

[23]Farkas [1997] for Hungarian, Farkas [2002] for Romanian, Yanovich [2005] for Russian, Henderson [2014] for Kaqchikel.

must be a pair of assignments in which x differs and v is fixed. Note also that our definition of the strict existential rules out branching. It follows that in a condition like (a), the variation requirement of non-specific indefinites cannot be satisfied. By defining a sentence as felicitous if it can be supported by an initial team, our analysis predicts the infelicity of (12).

Let us examine what happens when an operator (e.g., a universal quantifier) intervenes and licenses the non-specific indefinite: $\forall y \exists_s x(\phi(x, v) \wedge var(v, x))$. The universal quantifier leads to a universal y-extension of the initial team (b). In the extended team $var(v, x)$ can be then satisfied (c).

(a)

v
v_1
v_2

(b)

v	y
v_1	a_1
v_1	a_2
v_2	a_1
v_2	a_2

(c)

v	y	x
v_1	a_1	d_1
v_1	a_2	d_2
v_2	a_1	d_2
v_2	a_2	d_2

Table 11: Licensing of non-specific indefinites

We note that the non-specificity $var(v, x)$ atom is satisfied in teams like (c) in Table 11, even though the value of x does not vary in v_2. The reason for this is that $var(v, x)$ is satisfied as long as we can find an epistemic possibility where the value of the variable for the indefinite is not determined. We do believe that this partial variation, from an epistemic viewpoint, corresponds to the received empirical distribution of this class of indefinites. Using the stronger variation atom mentioned in footnote 11, would account for variation in all epistemic possibilities.

We also observe that other indefinites cannot license non-specific ones. For instance, an epistemic indefinite (with the $var(\varnothing, x)$ condition) for the initial team in (a) leads to an extension where the value of x is different in v_1 and v_2, but still $var(v, x)$ is not satisfied, since indefinites are strict existential and do not allow for branching extensions which are necessary to license non-specific indefinites.

At the beginning of this section, we have observed that indefinites which go under the name of *dependent* indefinites are not licensed by modals. In this framework, this restriction can be captured by assuming that dependent indefinites are associated with $var(v\vec{w}, x)$, where $\vec{w}$ is a possibly empty sequence of variables introduced in the discourse, ensuring that world variables are not sufficient to trigger variation.

Other operators, like negation or modals, can license non-specific indefinites. We will dedicate the next two section to extend our two-sorted team semantics framework with negation and modality.

Before we turn to negation, we point out that cross-linguistically not all non-specific indefinites are licensed by clausemate sentential negation, but they are acceptable in other NPI-licensing contexts, such as conditionals' antecedents. This problem is known in the literature as the Bagel problem [Pereltsvaig, 2004].

Pereltsvaig [2004] observed that for Russian *nibud'*, the anti-morphic context of clausemate sentential negation creates "a bagel hole" with respect to downward entailing environments in which *nibud'* is licensed. The reason for this is the presence of NPI lexical items that occur only in clausemate sentential negation and are preferred in such contexts due to lexical competition.

4.5 Negation

In this section, we will extend the two-sorted language presented in Section 3 with a general notion of negation. We will first explore different notions of negation which have been used and studied in dependence logic and team semantics. We will then focus on *intensional negation*, which we adopt in the present work.

An intuitive way to define negation in a team-based system would be as in Definition 17. We refer to it as classical or Boolean negation.

Definition 17 (Negation)
$M, T \models \neg_B \phi(v) \Leftrightarrow M, T \not\models \phi(v)$

An immediate consequence of Definition 17 is the failure of the Law of Excluded Middle, given our notion of split disjunction. More strikingly, Definition 17 seems to be ill-suited to model the negation of our dependence and variation atoms in the interaction with the existential. For instance, suppose that we are negating a specific known indefinite. In (14), we use for simplicity the English *a certain* to convey the meaning of a specific known indefinite (i.e., $dep(\varnothing, x)$), even though the empirical distribution of English *a certain* is different from the specific known indefinites we considered before. A sentence like (14) should come out as supported if there is a specific book which John does not have or, more strongly, if John does not have any book at all. However, $\neg_B(\exists x \phi(x, v) \wedge dep(\varnothing, x))$ is supported in all cases in which it is not the case that John does have a book and we know which one. So a team in which we do not know the value of x (i.e., a specific unknown reading) would be supporting. We therefore need a different way to deal with negation.[24]

(14) a. John does not have a certain book.
b. $\neg(\exists x \phi(x, v) \wedge dep(\varnothing, x))$

An alternative notion of negation is the so-called dual negation. For the classical fragment of our language which does not include dependence or variation atoms, this would amount to Definition 18.

Definition 18 (Dual Negation)
$M, T \models \neg_D \alpha(v) \Leftrightarrow \forall i \in T : M, \{i\} \not\models \alpha(v)$

[24]In general, adding a negation operator like in (Definition 17) greatly increases the expressive power of our logic, leading to full second-order logic and making the axiomatization more difficult (see Väänänen [2007a], Kontinen and Väänänen [2011]).

A natural question is how to extend Definition 18 for dependence atoms. The choice taken in the Dependence Logic tradition is to assume that a dependence atom is not supported only in the empty team. This makes it possible to preserve double negation elimination and the De Morgan's laws. However, it admittedly does not capture the failure of functional dependence, and it would again give the wrong predictions for cases like (14). In fact, provided that the team is non-empty, $\neg_D(\exists x \phi(x,v) \wedge dep(\emptyset, x))$ reduces to $\forall x \neg_D \phi(x,v)$, which is stronger than the intended reading.

Given our two-sorted framework, we adopt an intensional notion of negation [Brasoveanu and Farkas, 2011, Berto, 2015], which we define in Definition 19:

Definition 19 (Intensional Negation)
$\neg_I \phi(v) \Leftrightarrow \forall w(\phi(w) \rightarrow v \neq w)$

Definition 19 says that when ϕ does not hold in the actual world, it must be the case that for all worlds w in which ϕ holds, w must be different from the actual world.[25] Clearly, to properly interpret Definition 19, we need a semantic clause for implication. In Dependence Logics [Yang, 2014, Abramsky and Väänänen, 2009] different notions of implication have been studied (material, intuitionistic, linear and maximal). Here we adopt (a version of) the maximal implication, which as we will see gives the desired results:

Definition 20 (Implication)
*$M, T \models \phi \rightarrow \psi \Leftrightarrow$ for **some** $T' \subseteq T$ s.t. $M, T' \models \phi$ and T' is maximal, we have $M, T' \models \psi$*

Definition 21 (Maximal Team)
Given a model M and a formula ϕ, a team T maximally satisfies ϕ iff $M, T \models \phi$ and for all T'' s.t. $T' \subset T'' \subseteq T$, it holds $M, T'' \not\models \phi$

The semantic clause for implication says that a formula $\phi \rightarrow \psi$ holds when there is a maximal team which supports the antecedent and supports the consequent. An alternative definition of maximal implication requires 'all' maximal teams T to support the antecedent (not just 'some T'').

Preliminary, we do observe that for classical formulas (formulas without the dependence or the variation atom), the intensional notion of negation in Definition 19 and the dual negation in Definition 18 are equivalent.

Some remarks on the 'for some' versus 'for all' distinction in the definition of maximal implication are in order. For non-specific indefinites, the formula in the antecedent of the conditional will be union-closed, and thus this difference is trivialized. However, the *for some* clause will play a role when an atom like $dep(\emptyset, x)$ leads to more than one maximal supporting team (i.e., compatibly with different possible constants values for x.)

[25] Non-identity is defined as in the semantic clauses given at the beginning:

$$M, T \models x \neq y \Leftrightarrow \forall i \in T \text{ s.t. } i(x) \neq i(y)$$

Let's then consider cases like (15), again under the assumption that *a certain* stands for a specific known indefinite, triggering $dep(\varnothing, x)$:

(15) a. John does not have a certain book.

b. $\forall w(\exists_s x(\phi(x, w) \wedge dep(\varnothing, x)) \rightarrow v \neq w)$

The formula in (15b) should come out true when the initial team is $\{w_\varnothing\}$, corresponding to a world where John read no book), or $\{w_a\}$ (John read book a and not b) or $\{w_b\}$ (John read book b and not a). But not by $\{w_{ab}\}$, corresponding to a world where John read both book a and book b. This is precisely what we predict. When the initial team is $\{w_\varnothing\}$, both maximal teams satisfying the antecedent (i.e., $\exists_s x(\phi(x, w) \wedge dep(\varnothing, x))$) support the consequent (i.e., $v \neq w$). For $\{w_a\}$, we have two maximal teams satisfying the antecedent, but only the one which maps x to b also supports the consequent. For $\{w_{ab}\}$, none of the maximal teams satisfying the antecedent supports the consequent. We illustrate this in Table 12.

(a) Supporting Team

v	w	x
$w_\varnothing$	$w_\varnothing$	a
$w_\varnothing$	w_a	a
$w_\varnothing$	w_b	a
$w_\varnothing$	w_{ab}	a

(b) Non-Supporting Team

v	w	x
w_a	$w_\varnothing$	a
$\mathbf{w_a}$	$\mathbf{w_a}$	a
w_a	w_b	a
w_a	w_{ab}	a

(c) Non-Supporting Team

v	w	x
w_{ab}	$w_\varnothing$	a
w_{ab}	w_a	a
w_{ab}	w_b	a
$\mathbf{w_{ab}}$	$\mathbf{w_{ab}}$	$\mathbf{a}$

(d) Supporting Team

v	w	x
$w_\varnothing$	$w_\varnothing$	b
$w_\varnothing$	w_a	b
$w_\varnothing$	w_b	b
$w_\varnothing$	w_{ab}	b

(e) Supporting Team

v	w	x
w_a	$w_\varnothing$	b
w_a	w_a	b
w_a	w_b	b
w_a	w_{ab}	b

(f) Non-Supporting Team

v	w	x
w_{ab}	$w_\varnothing$	b
w_{ab}	w_a	b
w_{ab}	w_b	b
$\mathbf{w_{ab}}$	$\mathbf{w_{ab}}$	$\mathbf{b}$

Table 12: Worlds differ with respect to which books John has. In $w_\varnothing$ John has no book, in w_a John has only book a, and so on. The maximal teams satisfying the antecedent in (15b) are depicted in blue.

Let's now examine the interaction between non-specific indefinites and negation. As we will see, we predict that in this environment non-specific indefinites are licensed and behave like NPIs. To facilitate the analysis, we consider the example in (16), with 'some-*nibud*'' as a placeholder for a non-specific indefinite.

(16) a. John does not have some-*nibud*' book.

b. $\forall w(\exists_s x(\phi(x, w) \wedge var(v, x)) \rightarrow v \neq w)$.

Crucially, in this case, there is only one maximal team satisfying the antecedent. The variation atom $var(v, x)$ is trivialized, and the resulting reading is simply a negated existential, which is supported only for the initial team $\{w_{\emptyset}\}$.

(a) Supporting

v	w	x
$w_{\emptyset}$	$w_{\emptyset}$	a
$w_{\emptyset}$	w_a	a
$w_{\emptyset}$	w_b	b
$w_{\emptyset}$	w_{ab}	b

(b) Non-supporting

v	w	x
w_a	$w_{\emptyset}$	a
$\mathbf{w_a}$	$\mathbf{w_a}$	$\mathbf{a}$
w_a	w_b	b
w_a	w_{ab}	b

Table 13: Supporting and Non-supporting teams for (16b).

4.6 Modality

As noted at the end of Section 4.4, non-specific indefinites are licensed by modals. This occurs also for existential modals and not just universal ones, as illustrated in (17) for Russian *-nibud'*.

(17) a. *On kupil kakoj-nibud' to tort
He buy-PAST some-nibud cake.
'He bought a cake.'

b. On mog kupit' kakoj-nibud' tort
He can-PAST buy-INF some-nibud cake
'He could buy a cake.

The language of our logic is two-sorted, with also variables for worlds. We can therefore analyze modals as (lax) quantifiers over worlds ($\Diamond_w \sim \exists_{l(ax)} w; \Box_w \sim \forall w$). Necessity modals will be analyzed as universal quantifiers over worlds, and existential/possibility modals as lax existential quantifiers over worlds. Lax quantification allows for branching extensions and thus captures the availability of non-specific indefinites under possibility modals.

To see this, consider the basic paradigms in (18) and (19). The case of necessity modals parallels the case of universal quantifiers we have examined in the previous section. Existentials modals allow for lax functional extensions (i.e., they can lead to branching) and thus $var(v, x)$ can be satisfied as well.

(18) Necessity Modal
a. You must take some-*nibud* book
b. $\forall w \exists_s x(\phi \wedge var(v, x))$

(19) Possibility Modal
a. You may take some-*nibud* book
b. $\exists_l w \exists_s x(\phi \wedge var(v, x))$

We have seen how this framework captures universal and existential modality. Kratzer [1986] and many others distinguish between two broad classes of modality: epistemic modals, compatible with what the speaker knows, and root/deontic modals, compatible with a set of circumstances or normative rules. The necessity modal *must*, for instance, can be used epistemically, as in 'Sue must be home', or deontically, as in 'Sue must pay a fine'.

One important feature of epistemic modals are so-called epistemic contradictions which arise in sentences of the form $(\neg\phi \wedge \Diamond\phi)$:

(20) # It is not raining, and it might be raining.

As said, epistemic modality is related to the epistemic state of the speaker. And crucially, in this system, we already have a way to characterize the epistemic state of the speaker: the designated variable for the actual world v. As a result, we would like epistemic modals to be restricted to worlds over which v ranges. Deontic modality, on the other hand, is related to particular normative rules or desires which do not necessarily coincide with the state of affairs in the actual world. As a result, we would like deontic modality to range over worlds compatible with such norms, but not necessarily worlds over which v ranges.

Recall that the underlying idea of the framework we developed here is that the dependences in the values of the variable introduced by an indefinite across different assignments helped us to model scopal and epistemic effects in indefinites. Similarly, the relationship between *world* variables can be used to model the difference between epistemic and deontic modality. Towards this goal, we introduce the notion of inclusion atoms, first discussed in Galliani [2012] and also studied in Yang [2014]:

Definition 22 (Inclusion Atom)
$M, T \models \vec{x} \subseteq \vec{y} \Leftrightarrow$ *for all* $i \in T$, *there is a* $j \in T : i(\vec{x}) = j(\vec{y})$

Intuitively, (22) says that the values of $\vec{y}$ are also values of $\vec{x}$. Clearly, when $\vec{x} \subseteq \vec{y}$ and $\vec{y} \subseteq \vec{x}$, it must be that $\vec{x} = \vec{y}$. We give some illustrations in Table 14. In Table 14, $x \subseteq y$ holds since any value for x (namely, d_1 and d_2) is also a value of y. Similarly, $xz \subseteq xy$ holds, since any value for xz (namely, d_1d_2 and d_2d_4) is also a value for xy. But it does hold that $y \subseteq x$, since for instance d_3 is not a value for x.

x	y	z
d_1	d_1	d_2
d_1	d_2	d_2
d_2	d_3	d_4
d_2	d_4	d_4

$\subseteq (x, y)$ ✓
$\subseteq (xz, xy)$ ✓
$\subseteq (y, x)$ ✗

Table 14: Illustration of inclusion atoms

Recall that epistemic modals range only over worlds compatible with the speaker epistemic state (the values of v). Thus, we propose that an epistemic modal which introduces a variable w also triggers the restriction that $w \subseteq v$. By contrast, deontic modals are relational, since for each world, different normative rules are possible. To illustrate this, consider the basic cases in (21) and (22):

(21) Epistemic Existential Modal
  a. John might be in Paris.
  b. $\exists_l w\ (\phi(w) \land w \subseteq v)$

(22) Deontic Existential Modal
  a. John is allowed to be in Paris.
  b. $\exists_l w\ (\phi(w) \land R(v, w))$

The table below displays some possible lax extensions for (21) and (22). For epistemic modality, the condition $w \subseteq v$ guarantees that the worlds introduced by the functional extension will always be a subset of the values for v. For deontic modals, as illustrated in the examples in Table 15, it might not be the case that every world has access to the same set of 'normative-valid' worlds, and thus a world-dependent accessibility relation is needed. In other words, we are here proposing that epistemic modals are global, since they globally look at the epistemic state encoded by v, while deontic modals are relational, in line with several accounts of epistemic and deontic modality.

(a)

v
v_1
v_2
v_3

(b)

v	w
v_1	v_1
v_1	v_2
v_2	v_1
v_2	v_2
v_3	v_1
v_3	v_2

(c)

v	w
v_1	w_1
v_1	w_2
v_2	w_1
v_2	w_1
v_3	w_3
v_3	w_4

Table 15: Epistemic and Deontic Modals

This treatment of epistemic modals readily captures epistemic contradictions like (20). Clearly, if a statement does not hold in the epistemic possibilities in v, then it will also not hold in the worlds introduced by an epistemic modal, since they are always a subset of the values of v. So a formula of the form $(\neg Pa \land \Diamond Pa)$ can never be satisfied.[26]

[26]We also note that inclusion atoms could be employed to model cross-referential or partitive constructions, like the one below:

(1) There were zebras$_x$ outside. Some$_y$ of them ($y \subseteq x$) were sleeping.

The possibles values for the second occurrence of 'zebras' must be included in the values for the first one. Of course, a proper treatment of such constructions would involve a theory of bare nouns, plurals and partitivity. We do not expand on this here, but we note that our framework is expressive enough to model such phenomena.

Lastly, the reader might feel that the addition of inclusion atoms could complexify the simple structure of the logical language introduced in Section 3. However, it turns out that these atoms are quite interestingly related to each other. For instance, Galliani [2012] observes that $var(x, y)$ holds only if $xz \subseteq xy$ for some $z \neq y$.[27]

4.7 Epistemic indefinites

In Section 2, we briefly discussed the class of indefinites called 'epistemic indefinites' (EIs). EIs are well-studied in the literature [Alonso-Ovalle and Menéndez-Benito, 2010, 2013, 2017, Aloni and Port, 2015] and they include Spanish *algún*, Italian *un qualche*, German *irgendein* and many more. In Section 2 we claimed that EIs were associated with specific unknown and non-specific uses, and they triggered the variation atom $var(\emptyset, x)$. In this section, we elaborate on these ideas and refine our theory.

There are two proprieties which are shared by all EIs and that thus any theory of EIs should account for. First, they generate an undefeasible ignorance inference in episodic contexts.[28] For instance, the 'namely' continuation in (23) combined with Italian *un qualche* results in oddity. A similar behaviour can be observed for German *irgendein* in (24).

(23) Maria ha sposato un qualche dottore (#cioè Ugo).
Maria has married un qualche doctor (#namely Ugo)
Maria married some doctor, namely Ugo.'

(24) Irgendein Student hat angerufen. #Rat mal wer?
some student has called. #guess who?
'Some (unknown) student called. #Guess who?

Second, EIs can display a co-variation reading when they are embedded under universal quantifiers or other quantificational operators. The sentence in (25) is an example for Spanish *algún*. (25) is compatible with a situation in which each professor dances with a different student.

(25) Todos los profesores están bailando con algún estudiante.
all the professors are dancing with algún student.
'Every professor is dancing with some student.'

Note that the 'ignorance reading' is still available for cases like (25), even though the reading is less salient. We illustrate this for the case of *irgendein*:

[27] z does not have to be a variable already introduced in the discourse. The statement means that if we know that $var(x, y)$ is the case, then we can always find a new variable z different from y such that $xz \subseteq xy$. The other direction holds if we adopt the stronger version of variation atom mentioned in Definition 13.

[28] In certain contexts, EIs can also give rise to what are known as indifference readings. In such cases, speakers may use an EI to signal that the identity of the referent is not relevant or important in the given context, even if they might know the referent's identity.

(26) Jeder Student hat irgendein Buch gelesen.
every student has irgendein book read

a. Ignorance: There is a particular book which every student read. The speaker does not know which one.
b. Co-variation: For every student x, there is a book y s.t. x read y.

We point out that these ignorance and co-variation readings really parallel the specific unknown and non-specific uses we have examined in the previous sections, where compatibility with both uses was captured by the variation atom $var(\varnothing, x)$. Let us start with ignorance inferences in episodic contexts.

Our account predicts that in episodic contexts like (27), $var(\varnothing, x)$ gives rise to the ignorance component of EIs: $var(\varnothing, x)$ ensures that the value of x is not constant across all epistemic possibilities (i.e., the speaker does not know the value of x).

(27) a. Maria ha sposato un qualche dottore.
Maria has married un qualche doctor.
Maria married some doctor.'

b. $\exists_s x(\phi(x, v) \wedge var(\varnothing, x))$

As said, when combined with other quantificational operators, EIs can give rise to co-variation/non-specific uses. The $var(\varnothing, x)$ atom readily explains the availability of cases like (26). The crucial fact is that the two readings reflect the different scope of the indefinite, which is handled by dependence atoms (see Section 4.1). Consider the example in (28) and the supporting teams in Table 16. The indefinite can receive both wide-scope, modelled by $dep(v, x)$, or narrow-scope, modelled by $dep(vy, x)$. When the indefinite receives wide-scope, $var(\varnothing, x)$ ensures that the value of x changes across different epistemic possibilities. When the indefinite receives narrow-scope, the value of x can vary with respect to y and so does not need to vary with respect to v. This explains the disappearance of the ignorance effect in co-variation readings.

(28) Jeder$_y$ Student hat irgendein$_x$ Buch gelesen.
every student has irgendein book read

a. SPECIFIC UNKNOWN: $\forall y \exists_s x(\phi(x, v) \wedge dep(v, x) \wedge var(\varnothing, x))$
b. NON-SPECIFIC: $\forall y \exists_s x(\phi(x, v) \wedge dep(vy, x) \wedge var(\varnothing, x))$

(a)

v
v_1
v_2

(b)

v	y	x
v_1	b_1	a_1
v_1	b_2	a_1
v_2	b_1	a_2
v_2	b_2	a_2

(c)

v	y	x
v_1	b_1	a_1
v_1	b_2	a_2
v_2	b_1	a_1
v_2	b_2	a_2

Table 16: (a) Initial team; (b) Specific unknown; (c) Non-specific (co-variation)

It is worth pointing out that previous approaches [most notably, Alonso-Ovalle and Menéndez-Benito, 2010, 2017] assumed that EIs trigger an anti-singleton constraint which requires the domain of the indefinite to contain more than one individual. Our variation condition shares the same underlying idea. However, unlike Alonso-Ovalle and Menéndez-Benito [2017], we do not derive the ignorance effect as an implicature, but as part of the meaning of the indefinite, which also explains its undefeasibility. Moreover, our framework integrates the non-specific or co-variation uses of EIs in a more general theory of indefinites and scope.

We have examined how our account predicts both ignorance and co-variation uses. An additional desiderata is the NPI behaviour of certain EIs when they occur in downward-entailing contexts.[29] The crucial feature is the disappearance of the ignorance component: (29) means that nobody answered any question, not that there is a particular question that nobody answered and the speaker does not know which one.[30]

(29) Niemand hat irgendeine Frage beantwortet.
Nobody has irgend-one question answered.
'Nobody answered any question.'

In Section 4.5, we have extended our original framework with an intensional notion of negation. It turns out that this notion predicts also the NPI behaviour of epistemic indefinites under negation. A schematic example is illustrated in (30), together with a supporting and a non-supporting team in Table 17. (30b) is supported only if the initial team is $\{w_\varnothing\}$ (i.e., John did not read any book). In the other cases, as in the (b) example in Table 17, the maximality of the antecedent makes $v \neq w$ false. Note again, similarly for what happened for non-specific indefinites, the maximal team supporting the antecedent of (30b) is unique.

(30) a. John does not have *irgend*-book.
b. $\forall w(\exists_s x(\phi(x,w) \wedge dep(vw,x) \wedge var(\varnothing,x)) \rightarrow v \neq w)$

v	w	x
$w_\varnothing$	$w_\varnothing$	a
$w_\varnothing$	w_a	a
$w_\varnothing$	w_b	b
$w_\varnothing$	w_{ab}	b

v	w	x
w_a	$w_\varnothing$	b
$\mathbf{w_a}$	$\mathbf{w_a}$	**a**
w_a	w_b	b
w_a	w_{ab}	a

Table 17: Worlds differ with respect to which books John read. In $w_\varnothing$ John read no book, in w_a John read only book a, and so on. The maximal team of the antecedent of (30) is depicted in blue.

[29]The range of downward-entailing contexts where EIs are allowed to occur varies a lot cross-linguistically. For instance, they normally cannot combine with sentential negation. The polarity status of EIs is indeed quite complex: see Gianollo [2019] for an interesting diachronic perspective.

[30]The latter ignorance reading is available only in a particular salient context when there is a specific unknown question that none has read. However, alternative expressions are preferred in such cases.

Finally, one last desiderata concerns the free choice behaviour of the German EIs *irgendein*. When stressed and under a modal, *irgendein* exhibits a so-called free choice reading. It should be said that this behaviour is attested only in *irgendein* and other EIs typically do not allow for such a reading.

(31) Mary muss IRGENDEINEN Arzt heiraten.
Mary must irgend-one doctor marry.
'Mary must marry a doctor, any doctor is a permissible option'.

To capture such reading, we generalize our variation atoms to model the degree of variation. So far, the only requirement that we imposed with variation was that the value of the relevant variable should differ in at least 2 assignments. It is possible to generalize to level k [Väänänen, 2022]:[31]

Definition 23 (Generalized Variation)
$var_n(\vec{x}, y) \Leftrightarrow$ *for all* $i \in T : |\{j(y) : j' \in T$ *and* $i(\vec{x}) = j(\vec{x})\}| \geq n$

In the case of ignorance effects in epistemic indefinites, a different k could indicate a different degree of ignorance with respect to the value of x. For instance, if k is the same of the cardinality of the domain D, then the speaker is completely ignorant with respect to the value of x.

The meaning associated with free choice is $var_{|D|}(v, x)$: in all epistemic possibilities of the speaker, every value for x is a possible option. Crucially, free choice readings arise when *irgendein* is stressed [Haspelmath, 1997, Aloni and Port, 2015] and we claim that the role of stress is precisely to strengthen the variation to level $|D|$:

(32) Mary musste$_w$ irgendeinen$_x$ Mann heiraten.
Mary had-to irgend-one man marry.

a. SPECIFIC UNKNOWN:
$\forall w \exists_s x\, (\phi \wedge dep(v, x) \wedge var_2(\emptyset, x))$

b. CO-VARIATION:
$\forall w \exists_s x\, (\phi \wedge dep(vw, x) \wedge var_2(\emptyset, x))$

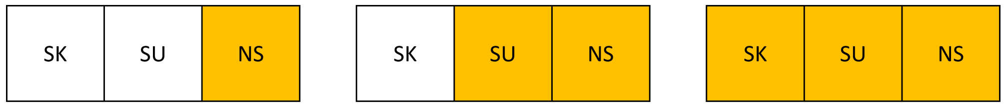
c. FREE CHOICE:
$\forall w \exists_s x\, (\phi \wedge dep(vw, x) \wedge var_{|D|}(v, x))$

Note that the variation condition needed to obtain free choice is v-variation $var_{|D|}(v, x)$, and not $var_{|D|}(\emptyset, x)$, which would be satisfied simply if we can find $|D|$- values of x across all epistemic possibilities of the speaker. This departs from our original minimal assumption that epistemic indefinites associate with

[31] The generalized variation atom in (23) is equivalent to:

$$\forall i \in T \exists j_1 \ldots \exists j_n \in T : \bigwedge_{1 \leq m \leq l \leq n} (i(\vec{x}) = j_m(\vec{x}) \text{ and } j_m(y) \neq j_l(y)$$

Note that this generalized atom is based on the stronger version of variation mentioned before. By requiring that $\exists i \in T$ instead of $\forall i \in T$, we can obtain the generalized version of the weaker variation atom we originally considered.



(a) Non-specific: $var(v, x)$ (b) Epistemic: $var(\emptyset, x)$ (c) Unmarked

Figure 6: Weakening of indefinites

$var(\emptyset, x)$. However, we note that among EIs, the *irgend*-series is the only one which displays free choice readings for which this additional requirement is needed.[32]

Finally, an important difference to note is that the free choice readings of *irgend*- are generally licensed by deontic modals, rather than epistemic ones. In our framework, epistemic modals are subject to a restriction in the form of an inclusion atom $w \subseteq v$, which ensures that epistemic modals range over possible values for the actual world. It is noteworthy that *irgend*-, when used under its SU reading, is typically associated with partial variation (i.e. $k < |D|$) as the speaker is generally not completely ignorant about all possible values for the referent of the indefinite. However, a free choice reading under an epistemic modal would require the speaker to be in a state of total epistemic variation, which is not the typical context for the use of *irgend*-, suggesting why deontic modals are the typical licensor of free choice readings for *irgend*-.

4.8 Weakening & Semantic Change

In this section, we consider some diachronic pathways of indefinites and their relationship with the formal system discussed in this paper. Cross-linguistically, we witness a general tendency of non-specific indefinites to acquire SU uses, turning into epistemic indefinites (the path from (a) to (b) in Figure 6). This occurred for instance for French *quelque* [Foulet, 1919] and German *irgendein* [Port and Aloni, 2015].

Haspelmath [1997] proposed that indefinites gradually acquire new functions on his map (see Figure 1) from the right (non-specific) region to the left (specific) region due to weakening (an indefinite gets a new function, and it thus becomes weaker than the previous form). This would explain the cases from (a) to (b) mentioned above. However, we do not witness further weakening triggering the acquisition of SK (i.e., from (b) to (c)).[33]

Our framework makes the notion of weakening precise in terms of logical entailment between atoms. This can be also illustred in the Dependence Square of Opposition, discussed in Section 4.3 and represented again below for ease of

[32]In general, we suggest this treatment of free choice is line with the distribution of other so-called existential free choice items [Chierchia, 2013].

[33]Haspelmath [1997] claims that this occurred for Portuguese *algum*. The data however suggests that *algum* is still an epistemic indefinite and SK uses are not allowed. See Gianollo [2020] for an interesting analysis of the Romance descendants of Latin *aliquis*.

illustration, where entailment corresponds to subalternation. This leads to the following two predictions:

1. Non-specific ($var(v, x)$) > Epistemic ($var(\emptyset, x)$);
2. Specific-known ($dep(\emptyset, x)$) > Specific ($dep(v, x)$).

The former corresponds to the diachronic path of the epistemic indefinites outlined above. The latter is a path predicted by the weakening mechanism, which could be operational in the development of indefinite articles from one-cardinals [Givón, 1981], but it is admittedly not attested in the domain of indefinite pronouns. Quite importantly, we have no further 'atomic weakening' triggering the acquisition of sk, which explains why such development is not attested.[34]

Another perspective on possible constraints in language change is that the representation of *known* versus *unknown* requires variables ranging over a domain of *abstract* entities, which typically occur later in grammaticalization processes [Traugott and Dasher, 2002, Heine, 1997].

While in a language without world variables, the contrast between $dep(\emptyset, x)$ and $var(\emptyset, x)$ accounts for the difference between specific and non-specific, the use of world variables is necessary to express also the known vs unknown distinction (with now $var(\emptyset, x)$ vs $dep(\emptyset, x)$ standing for unknown vs known and $var(v, x)$ and $dep(v, x)$ standing for specific and non-specific). Assuming thus that individual quantification precedes world quantification leads to the following two predictions:

1. Non-specific > Epistemic;
2. Specific > Specific known.

We observe that the path from non-specific to epistemic can be explained by both the weakening and concreteness factors, which can be taken as evidence of why this diachronic path is common. Furthermore, we can hypothesize that the absence of a change from specific-known to specific as a form of weakening can be explained by its clash with the second constraint.

4.9 Specific Indefinites

Finally, we dedicate this section to some remarks on the behaviour of specific indefinites and scope. The aim of the present paper is to characterize the behaviour of indefinites from a typological and cross-linguistic viewpoint. As a result, so far we have not provided extended discussions of the comprehensive behaviour of a particular indefinite in a given language.

In the previous sections, we have argued that specific indefinites (specific or specific known) are only compatible with wide-scope readings. Recently,

[34]To get unmarked indefinites from epistemic ones, we would need $var(\emptyset, x) \vee dep(\emptyset, x)$, which trivializes the dependence conditions, and it is arguably a complex operation. Note also that $var(\emptyset, x) \wedge dep(\emptyset, x) \models \bot$, which shows that sk contradicts the atom for epistemic indefinites.

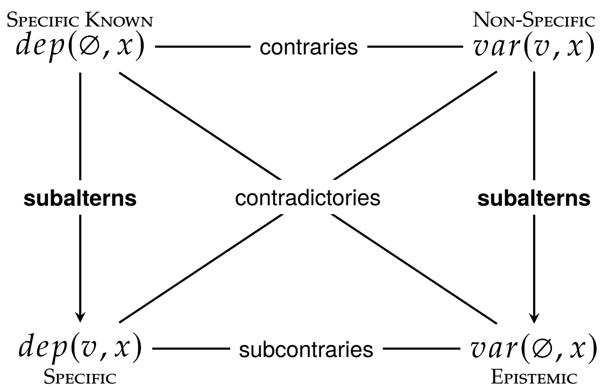


Figure 7: Dependence Square of Opposition

Martí and Ionin [2019] discussed the scopal behaviour of Russian *koe* and *to* in a series of experiments. To remind the reader, in our system *koe* is classified as a specific known indefinite, triggering $dep(\varnothing, x)$, while *to* is an epistemic indefinite, triggering $var(\varnothing, x)$. Martí and Ionin [2019] examined which readings are available by paraphrasing a sentence containing an indefinite with an intended interpretation and asking if such reading would be possible. (33) is an example. With regard to *koe*, they showed that the default reading for cases like (33) is indeed wide-scope. But narrow-scope configurations are allowed when the reading is functional, as in (33b), but crucially not when such function is not made explicit, as in (33c).

(33) Každyj doktor osmotrel koe-kakogo pacienta.
every doctor examined koe-wh patient
'Every doctor examined some patient.'

a. WSR context (no function supported):

Točnee, vse doktora osmotreli pacienta, kotoryj privlek
more precisely all doctors examined patient which attracted
vseobščee vnimanie svoimi neobyčnymi simptomami.
everyone's attention self's unusual symptoms

'That is, all the doctors examined the patient who attracted everyone's attention with his unusual symptoms.'

b. functional NSR context (function supported):

Točnee, každyj doktor osmotrel samogo bol'nogo pacienta
more precisely every doctor examined most sick patient
v ego otdelenii.
in his unit.

'That is, every doctor examined the sickest patient in his unit.'

c. NSR (no function supported):

Točnee,	vse	doktora	osmotreli	raznyh	pacientov.
more precisely	all	doctors	examined	different	patients

'That is, all the doctors examined different patients.'

(from Martí and Ionin [2019])

This would explain the contrast between a quantifier like *every doctor* which allows for functional readings with respect to the doctor and the indefinite, and an attitude verb like *want* which does not allow for such functional readings. In our framework, such functional readings can be captured by introducing the relevant choice functional mechanism in line with previous choice-functional approaches of indefinites (e.g., Reinhart [1997], Kratzer [1998], Winter [1997]). A tentative logical form for a functional reading like (33b) is offered in (34), where f is a function from doctors to units, and the dependence atom now guarantees that the function remains the same across all the assignments (i.e., it is possible to associate the narrow-scope reading with a specific function). Note that such analysis makes use of variables ranging over functions, which are not part of the logical language we introduced. We do not pursue this any further, since our account of these functional readings would not offer any novel result besides what choice-functional analyses have already considered.

(34) $\forall y \exists f(\phi(f(y), v) \wedge dep(f, \varnothing))$

Martí and Ionin [2019] discuss also similar cases for intermediate scope configurations, and they show that *koe* does indeed licence such readings, when they are interpreted functionally. Moreover, they show that *to* is compatible with all readings, both functional and non-functional/quantificational, confirming our proposal that an indefinite like *to* does not trigger any restriction with respect to the dep atoms.

4.10 Final Proposal & Illustration

Let us recap what we have discussed so far. Indefinites are strict existentials, which are interpreted *in-situ*. The scope of indefinites is accounted by dependence atoms, which allow co-variation with all the variables in the syntactic scope of the indefinite (see generalization in (7)). Marked indefinites further trigger the obligatory activation of particular dependence or variation atoms:

(35) Marked Indefinites & Atoms

$$O_{z_1} \dots O_{z_n} \exists_s x(\phi \wedge ATOM)$$

a. Plain: $dep(\vec{y}, x)$, where $\vec{y} \subseteq v\vec{z}$
b. Specific Known: $dep(\vec{y}, x)$ with $\vec{y} = \varnothing$
c. Specific: $dep(\vec{y}, x)$ with $\vec{y} = v$
d. Epistemic: $dep(\vec{y}, x) \wedge var(\vec{z}, x)$ with $\vec{z} = \varnothing$

	WS-K $dep(\emptyset, x)$	WS-U $dep(v, x)$	IS $dep(vy, x)$	NS $dep(vyz, x)$
unmarked	✓	✓	✓	✓
specific $dep(\subseteq v, x)$	✓	✓	✗	✗
non-specific $var(v, x)$	✗	✗	✓	✓
epistemic $var(\emptyset, x)$	✗	✓	✓	✓
specific known $dep(\emptyset, x)$	✓	✗	✗	✗
specific unknown $dep(v, x) \wedge var(\emptyset, x)$	✗	✓	✗	✗

Table 18: Marked Indefinites & Scope

e. Non-specific: $dep(\vec{y}, x) \wedge var(\vec{z}, x)$ with $\vec{z} = v$

f. Specific Unknown: $dep(\vec{y}, x) \wedge var(\vec{z}, x)$ with $\vec{y} = v$ and $\vec{z} = \emptyset$

As an illustration, consider a configuration of the form $\forall z \forall y \exists_s x \phi$, where the existential stands for a (marked) indefinite. Our predictions are summarized in Table 18. We predict wide-scope (known and unknown) for specific indefinites. Epistemic indefinites allow all scope configurations, except for wide-scope with known referent. For non-specific indefinites, we predict that they do not allow for wide-scope readings, but they admit other readings. This is explained by the fact that non-specific indefinites need at least one operator with whom they can co-vary. This behaviour of non-specific indefinites is coherent with the data of Russian *-nibud'* discussed in [Partee, 2004], as illustrated in (36). Moreover, in Section 4.9 we have already discussed in which sense our predictions match the experimental results for the scope of specific and epistemic indefinites.

(36) Možet byt', Maša xočet kupit' kakuju-nibud' knigu.
may be, Maša want buy which-INDEF. book.

a. Narrow Scope: It may be that Maša wants to buy some book.

b. Intermediate Scope: It may be that there is some book which Maša wants to buy.

c. #Wide-scope: There is some book such that it may be that Maša wants to buy it.

5 Conclusions

We have developed a two-sorted team semantics framework accounting for indefinites. In this framework, marked indefinites trigger the obligatoriness of dependence or variation atoms, responsible for their scopal and epistemic interpretations. We have applied the framework to characterize the typological variety of indefinites in the case of (non-)specificity. We have then showed how this system accounts for several properties and phenomena associated with (non-)specific indefinites.

Future studies could focus on further exploring the logical properties of the framework presented here. Additionally, it would be valuable to extend the account to include other functional uses of indefinites, plurals and plural indefinites, and within-language analyses of particular indefinite systems.

References

S. Abramsky and J. Väänänen. From if to bi. *Synthese*, 167(2):207–230, 2009.

M. Aloni. *Quantification under Conceptual Covers*. PhD thesis, ILLC, University of Amsterdam, 2001. URL `https://hdl.handle.net/11245/1.173339`.

M. Aloni. Logic and conversation: the case of free choice. *Semantics and Pragmatics*, 15(5), 2022. URL `https://doi.org/10.3765/sp.15.5`.

M. Aloni and A. Port. Epistemic indefinites and methods of identification. *Epistemic indefinites: Exploring modality beyond the verbal domain*, pages 117–140, 2015. doi: 10.1093/acprof:oso/9780199665297.003.0006.

L. Alonso-Ovalle and P. Menéndez-Benito. Modal indefinites. *Natural Language Semantics*, 18(1):1–31, 2010. URL `https://doi.org/10.1007/s11050-009-9048-4`.

L. Alonso-Ovalle and P. Menéndez-Benito. Two views on epistemic indefinites. *Language and Linguistics Compass*, 7(2):105–122, 2013.

L. Alonso-Ovalle and P. Menéndez-Benito. Epistemic indefinites: on the content and distribution of the epistemic component. *Modality Across Syntactic Categories*, pages 11–29, 2017. URL `https://doi.org/10.1093/acprof:oso/9780198718208.001.0001`.

F. Berto. A modality called 'negation'. *Mind*, 124(495):761–793, 2015.

D. Bhat. Indefinite pronouns in kannada. In *The Life of Language*, pages 369–384. De Gruyter Mouton, 2011.

A. Brasoveanu and D. Farkas. How indefinites choose their scope. *Linguistics and Philosophy*, 34(1):1–55, 2011. URL `https://doi.org/10.1007/s10988-011-9092-7`.

L. Champollion, J. Bledin, and H. Li. Rigid and Flexible Quantification in Plural Predicate Logic. In *Semantics and Linguistic Theory*, volume 27, pages 418–437, 2017. URL `https://doi.org/10.3765/salt.v27i0.4190`.

S. Charlow. The scope of alternatives: Indefiniteness and islands. *Linguistics and Philosophy*, 43(4):427–472, 2020. URL `https://doi.org/10.1007/s10988-019-09278-3`.

G. Chierchia. *Logic in grammar: Polarity, free choice, and intervention*. OUP Oxford, 2013. doi: 10.1093/acprof:oso/9780199697977.001.0001.

P. J. Dekker. *Transsentential meditations: Ups and downs in dynamic semantics*. PhD thesis, ILLC, University of Amsterdam, 1993. URL `https://hdl.handle.net/11245/1.392554`.

É. Enguehard and E. Chemla. Connectedness as a constraint on exhaustification. *Linguistics and Philosophy*, 44(1):79–112, 2021. URL https://doi.org/10.1007/s10988-019-09286-3.

D. Farkas. Quantifier scope and syntactic islands. In *Papers from the Seventeenth Regional Meeting of the Chicago Linguistic Society*, number 17, pages 59–66, 1981. URL https://www.researchgate.net/publication/243752746_Quantifier_scope_and_syntactic_islands.

D. Farkas. Specificity and Scope. In L. Nash and G. Tsoulas, editors, *Langues et Grammaire 1*, pages 119–137. 1994.

D. Farkas. Dependent indefinites. empirical issues in formal syntax and semantics, ed. f. corblin et al., 243-267, 1997.

D. Farkas. Extreme non-specificity in romanian. *Amsterdam Studies in the Theory and History of Linguistic Science. Series 4*, pages 127–152, 2002.

D. Farkas and A. Brasoveanu. Kinds of (Non)Specificity. *The Wiley Blackwell Companion to Semantics*, pages 1–26, 2020.

J. D. Fodor and I. A. Sag. Referential and quantificational indefinites. *Linguistics and Philosophy*, 5(3):355–398, 1982. URL https://doi.org/10.1007/BF00351459.

L. Foulet. Étude de syntaxe française: Quelque. *Romania*, 45(178), 1919. URL https://doi.org/10.3406/roma.1919.5158.

P. Galliani. *The dynamics of imperfect information*. PhD thesis, ILLC, University of Amsterdam, 2012. URL https://hdl.handle.net/11245/1.377053.

P. Galliani. Dependence Logic. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Summer 2021 edition, 2021. URL https://plato.stanford.edu/archives/sum2021/entries/logic-dependence/.

P. Gardenfors. The Geometry of Meaning: Semantics Based on Conceptual Spaces, 2014. URL https://doi.org/10.7551/mitpress/9629.001.0001.

C. Gianollo. *Indefinites between Latin and Romance*, volume 33. Oxford University Press, 2019.

C. Gianollo. DP-internal Inversion and Negative Polarity: Latin *aliquis* and its Romance Descendants. *Probus*, 32(2):271–302, 2020. URL https://doi.org/10.1515/probus-2020-0005.

T. Givón. On the development of the numeral 'one'as an indefinite marker. *Folia Linguistica Historica*, 15(Historica-vol-2-1):35–54, 1981. doi: 10.1515/flih.1981.2.1.35.

J. Groenendijk and M. Stokhof. Dynamic Predicate Logic. *Linguistics and Philosophy*, 14(1):39–100, 1991. URL https://doi.org/10.1007/BF00628304.

M. Haspelmath. Indefinite pronouns, 1997. URL https://doi.org/10.1093/oso/9780198235606.001.0001.

B. Heine. *Cognitive foundations of grammar*. Oxford University Press, 1997.

R. Henderson. Dependent indefinites and their post-suppositions. *Semantics and Pragmatics*, 7:6–1, 2014.

W. Hodges. Compositional semantics for a language of imperfect information. *Logic Journal of the IGPL*, 5(4):539–563, 1997. URL https://doi.org/10.1093/jigpal/5.4.539.

H. Kamp and Á. Bende-Farkas. Epistemic specificity from a communication-theoretic perspective. *Journal of Semantics*, 36(1):1–51, 2019. URL https://doi.org/10.1093/jos/ffy005.

J. Kontinen and J. Väänänen. A remark on negation in dependence logic. *Notre Dame Journal of Formal Logic*, 52(1):55–65, 2011.

A. Kratzer. Conditionals. *Chicago Linguistics Society*, 22(2):1–15, 1986.

A. Kratzer. Scope or Pseudoscope? Are there Wide-Scope Indefinites? In *Events and grammar*, pages 163–196. Springer, 1998. URL https://doi.org/10.1007/978-94-011-3969-4_8.

M. Lück. *Team logic: axioms, expressiveness, complexity*. PhD thesis, Hannover: Institutionelles Repositorium der Leibniz Universität Hannover, 2020. URL https://doi.org/10.15488/9376.

L. Martí and T. Ionin. Wide scope indefinites in russian: an experimental investigation. *Glossa: a journal of general linguistics*, 4(1), 2019.

B. H. Partee. Semantic typology of indefinites ii, 2004. URL https://people.umass.edu/partee/RGGU_2004/RGGU0411annotated.pdf. Lecture Notes RGGU 2004.

A. Pereltsvaig. Negative polarity items in russian and the 'bagel problem'. *Negation in Slavic*, 153:178, 2004.

A. Port and M. Aloni. The diachronic development of German *Irgend*-indefinites, 2015. URL https://www.marialoni.org/resources/AloniPort21.pdf. Ms, University of Amsterdam.

T. Reinhart. Quantifier Scope: How labor is Divided Between QR and Choice Functions. *Linguistics and Philosophy*, 20(4):335–397, 1997. URL https://doi.org/10.1023/A:1005349801431.

R. Schwarzschild. Singleton Indefinites. *Journal of semantics*, 19(3):289–314, 2002. URL `https://doi.org/10.1093/jos/19.3.289`.

S. Srinivas and K. Rawlins. On the indefinite readings of kannada bare nominals. In *37th West Coast Conference on Formal Linguistics*, pages 214–225. Cascadilla Proceedings Project, 2021.

S. Steinert-Threlkeld, N. Imel, and Q. Guo. A semantic universal for modality. *Semantics and Pragmatics*, 16:1–EA, 2023.

E. C. Traugott and R. B. Dasher. *Regularity in Semantic Change*. Cambridge Studies in Linguistics. Cambridge University Press, 2002. doi: 10.1017/CBO9780511486500.

J. Väänänen. Dependence logic: A new approach to independence friendly logic, 2007a. URL `https://doi.org/10.1017/CBO9780511611193`.

J. Väänänen. Team Logic. In *Interactive logic. Selected papers from the 7th Augustus de Morgan workshop*, pages 281–302. Amsterdam University Press, 2007b. URL `https://www.jstor.org/stable/j.ctt45kdbf.14`.

J. Väänänen. An atom's worth of anonymity. *Logic Journal of the IGPL*, 2022. doi: 10.1093/jigpal/jzac074.

Y. Winter. Choice Functions and the Scopal Semantics of Indefinites. *Linguistics and Philosophy*, 20(4):399–467, 1997. URL `https://doi.org/10.1023/A:1005354323136`.

F. Yang. *On Extensions and Variants of Dependence Logic*. PhD thesis, University of Helsinki, 2014. URL `http://www.math.helsinki.fi/logic/people/fan.yang/dissertation_fyang.pdf`.

F. Yang. Propositional union closed team logics. *Annals of Pure and Applied Logic*, 173(6), 2022. doi: https://doi.org/10.1016/j.apal.2022.103102.

F. Yang and J. Väänänen. Propositional team logics. *Annals of Pure and Applied Logic*, 168(7):1406–1441, 2017. URL `https://doi.org/10.1016/j.apal.2017.01.007`.

I. Yanovich. Choice-functional series of indefinite pronouns and hamblin semantics. In *Semantics and linguistic theory*, volume 15, pages 309–326, 2005.