Meaning, Reference and Modality
Exercises 1-2*

Frege (1892)

The skeptic objection

Frege (1892, pp. 31 – 32) discusses an objection which skeptics might raise. In what does it consist? How does Frege try to bypass the problem?

I told you (that it was true)

Explain in which sense for Frege (1892, pp. 34 – 35) the sentence 'it is true that 5 is a prime number' is equivalent to '5 is a prime number'. What about 'it is true that it is true that it is true that 5 is a prime number'?

Optional: Consider uses of the word 'true' as in 'what the Pope says is true'. How, if so at all, can they be related to what Frege is discussing?

Subordinate sentences

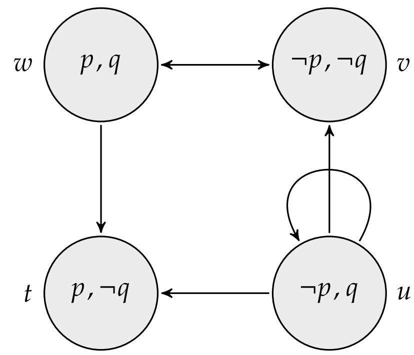
Explain why according to Frege (1892, pp. 36 – 37), the *Bedeutung* of subordinate sentences (introduced by 'that') is not a truth value.

Modal Propositional Logic

A model

Consider the model depicted in the picture below:

*For any question or comment, please contact Marco at `m.degano@uva.nl`



Show that:

1. $M, t \models \Box(p \wedge \neg p)$
2. $M, w \models \Diamond\neg p \to \Box\Box\Diamond\neg q$
3. $M, v \models \Box\Box\Box(p \leftrightarrow q)$
4. $M \models \neg p \leftrightarrow \neg\Box\neg q$
5. $M \models \Box p \vee \Diamond\neg q$
6. $M \models q \leftrightarrow \Diamond\Box p$

To know or not to know?

Suppose that $\Box$ stands for 'it is known that' and $\Diamond$ for 'it is conceived possible that'.

Consider the following intuitive principles:

1. $\Box p \to p$ 'Knowledge implies truth.'
2. $p \to \Diamond\Box p$ 'All truths are conceived knowable.'
3. $\Box(p \wedge q) \to \Box p \wedge \Box q$
   'Knowing a conjunction implies knowing each of the conjuncts.'
4. $\models \neg p \implies \models \neg\Diamond p$
   'If p can be proven false without assumptions, then p is not conceived possible.'

Consider now the following:

5. $p \wedge \neg\Box p$
6. $p \to \Box p$

Question 1: Explain in plain English what 5 and 6 mean.

Question 2: Assume that 1 – 4 are valid. Show that 6 holds. How do you make sense of this result?

Hint: Start by deriving that $\neg\Box(p \wedge \neg\Box p)$