Meaning, Reference and Modality Exercises 9-10-11*

Dynamic Semantics

DPL

Write out the DPL interpretation for the following pairs of formulas. Which pairs are equivalent?

- (1) a. $\exists x (Px \land Qx) \land Rx$ b. $\exists x (Px \land Qx \land Rx)$
- (2) a. $\exists x (Px \land Qx) \land Rx$ b. $\exists y (Py \land Qy) \land Rx$
- (3) a. $Rx \wedge \exists x (Px \wedge Qx)$ b. $Rx \wedge \exists y (Py \wedge Qy)$
- (4) a. $\neg \exists x P x \lor Q x$ b. $\exists x P x \to Q x$
- (5) a. $\exists x P x \land Q x$ b. $\neg (\exists x P x \rightarrow \neg Q x)$

Among the ones which are not-equivalent, which ones are s-equivalent? (Definition 7, Groenendijk & Stockhof 1991, p. 16)

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Update Semantics

Consider the formulas below. Are they valid in Veltman's update semantics?

- (6) a. $\Diamond p \to p$ b. $p \to \Diamond p$
- (7) a. $\Box p \to p$

b. $p \rightarrow \Box p$

A formula ϕ is valid iff $\forall s : s \subseteq s[\phi]$

 $s[\phi \rightarrow \psi] = \{i \in s | \text{ if } i \in s[\phi] \text{ then } i \in s[\phi][\psi]\}$

Dynamic Modal Predicate Logic

The Broken Vase (review)

Consider the broken vase scenario discussed in Groenendijk, Stockhof and Veltman (1996):

- (8) a. $\exists x H x \land \Diamond G x$
 - b. $\exists x(Hx \land \Diamond Gx)$

(6a) and (6b) are not equivalent, given GSV (1996)'s treatment of $\exists x$ as in (A) below. Consider now the global assignment in (B), and discuss the consequences for the broken vase scenario.

(A)
$$s[\exists x\phi] = \bigcup_{d \in D} (s[x/d][\phi])$$

(B)
$$s[\exists x\phi] = (\bigcup_{d \in D} s[x/d])[\phi]$$

Consistent and Coherent

Consider the sequence of sentences below. Treat $\exists x P x$ with a uniqueness requirement $\exists ! x P x$. Are they *consistent*? Are they *coherent*? Do the results match your intuitions?

(9) a. Someone has done it. It might be Alice. But it also might not be Alice.

b. $\exists x P x \land \Diamond(x = a) \land \Diamond(x \neq a)$

(10) a. Someone has done it. It might not be Alice. It is Alice

b. $\exists x P x \land \Diamond (x \neq a) \land (x = a)$

(11) a. Someone has done and it might be Alice and it might not be Alice.

b. $\exists x (Px \land \Diamond (x = a) \land \Diamond (x \neq a))$

(12) a. Someone has done it. Alice has done it. Anyone might be Alice. Bob might have done it.

b. $\exists x P x \land (x = a) \land \forall x (\diamond (x = a)) \land \diamond (x = b)$

Now drop the uniqueness requirement $\exists !xPx$ and treat $\exists xPx$ as $\exists xPx$. Which ones are now *coherent*? Which ones are now *consistent*?

Definitions

Dynamic Predicate Logic DPL

Language of predicate logic with identity. First order models $M = \langle D, I \rangle$

Formulas **denote** sets of assignment pairs:

$$\begin{split} \llbracket Rt_1 \dots t_n \rrbracket &= \{ \langle g, h \rangle \mid h = g \And \langle \llbracket t_1 \rrbracket_h, \dots, \llbracket t_n \rrbracket_h \rangle \in I(R) \} \\ \llbracket t_1 = t_2 \rrbracket &= \{ \langle g, h \rangle \mid h = g \And \llbracket t_1 \rrbracket_h = \llbracket t_2 \rrbracket_h \} \\ \llbracket \neg \phi \rrbracket &= \{ \langle g, h \rangle \mid h = g \And \neg \exists k : \langle h, k \rangle \in \llbracket \phi \rrbracket \} \\ \llbracket \phi \land \psi \rrbracket &= \{ \langle g, h \rangle \mid \exists k : \langle g, k \rangle \in \llbracket \phi \rrbracket \And \langle k, h \rangle \in \llbracket \psi \rrbracket \} \\ \llbracket \phi \lor \psi \rrbracket &= \{ \langle g, h \rangle \mid \exists k : \langle g, k \rangle \in \llbracket \phi \rrbracket \And \langle k, h \rangle \in \llbracket \psi \rrbracket \} \\ \llbracket \phi \to \psi \rrbracket &= \{ \langle g, h \rangle \mid h = g \And \exists k : \langle h, k \rangle \in \llbracket \phi \rrbracket \lor \langle h, k \rangle \in \llbracket \psi \rrbracket \} \\ \llbracket \phi \to \psi \rrbracket &= \{ \langle g, h \rangle \mid h = g \And \forall k : \langle h, k \rangle \in \llbracket \phi \rrbracket \Rightarrow \exists j : \langle k, j \rangle \in \llbracket \psi \rrbracket \} \\ \llbracket \exists x \phi \rrbracket &= \{ \langle g, h \rangle \mid h = g \And \forall k : k[x]g \And \langle k, h \rangle \in \llbracket \phi \rrbracket \} \\ \llbracket \forall x \phi \rrbracket &= \{ \langle g, h \rangle \mid h = g \And \forall k : k[x]h \Rightarrow \exists j : \langle k, j \rangle \in \llbracket \phi \rrbracket \} \end{split}$$

Equivalence: $\phi \equiv \psi$ iff $\forall M : [\![\phi]\!]_M = [\![\psi]\!]_M$ (same denotation)

Satisfaction set: $|\phi|_M^s = \{g \mid \exists h : \langle g, h \rangle \in \llbracket \phi \rrbracket_M \}$

Production set: $|\phi|_M^p = \{h \mid \exists g : \langle g, h \rangle \in \llbracket \phi \rrbracket_M \}$

s-Equivalence: $\phi \equiv_s \psi$ iff $\forall M : |\phi|_M^s = |\psi|_M^s$ (same satisfaction set)

p-Equivalence: $\phi \equiv_p \psi$ iff $\forall M : |\phi|_M^p = |\psi|_M^p$ (same production set)

Entailment:

$$\phi_1, \dots, \phi_n \models \psi \text{ iff } \forall M \forall g \forall h : \langle g, h \rangle \in \llbracket \phi_1 \land \dots \land \phi_n \rrbracket_M \Longrightarrow \exists k : \langle h, k \rangle \in \llbracket \psi \rrbracket_M$$
$$\phi_1, \dots, \phi_n \models \psi \text{ iff } \forall M : |\phi_1 \land \dots \land \phi_n|_M^p \subseteq |\psi|_M^s$$

Update Semantics US

Information state *s*: set of valuations (called possibilities *i*).

Interpretation is an **update function** over information states.

$$\begin{split} s\llbracket p \rrbracket &= \{i \in s \mid i(p) = 1\} \\ s\llbracket \neg \phi \rrbracket = s - s\llbracket \phi \rrbracket \\ s\llbracket \phi \land \psi \rrbracket = s\llbracket \phi \rrbracket \llbracket \psi \rrbracket \\ s\llbracket \phi \rightarrow \psi \rrbracket = \{i \in s \mid \text{if } i \in s\llbracket \phi \rrbracket \text{ then } i \in s\llbracket \phi \rrbracket \llbracket \psi \rrbracket \} \\ s\llbracket \phi \phi \rrbracket = \{i \in s \mid s\llbracket \phi \rrbracket \neq \emptyset \} \\ s\llbracket \neg \phi \rrbracket = \{i \in s \mid s\llbracket \phi \rrbracket \neq \emptyset \} \\ s\llbracket \neg \phi \rrbracket = \{i \in s \mid s \subseteq s\llbracket \phi \rrbracket \} \end{split}$$

Validity: ϕ is valid iff $\forall s : s \subseteq s[\![\phi]\!]$

Dynamic Modal Predicate Logic DMPL

Possibility *i*: triple $i = \langle r, g, w \rangle$ based on a set of individuals *D* and a set of worlds (interpretation functions) *W*, with *r* a referent system (injection from a set of variables into a set of numbers); *g* an assignment function from the range of *r* into *D*; $w \in W$ an interpretation function.

Information State *s*: set of possibilities *s* s.t. $\forall i, i' \in s : i$ and *i'* have the same referent system.

Extension $i \le i'$: a possibility *i* extends into a possibility i', $i \le i'$, iff $r \le r'$ and $g \subseteq g'$ and w = w'.

Information State Update $s \le s'$: an information state s' is an update of state $s, s \le s'$, iff $\forall i' \in s' \exists i \in s$: $i \le i'$

Given a possibility $i = \langle r, g, w \rangle$ based on *D* and *W* with *Var* the domain of *r*, $i(\alpha)$ is defined as

 $i(\alpha) = w(\alpha) \in D$ if α is an individual constant $i(\alpha) = d$ if α is $this_d$

 $i(\alpha) = w(\alpha) \subseteq D^n$ if α is a *n*-ary predicate

 $i(\alpha) = g(r(\alpha)) \in D$ if α is a variable $v \in Var$

Interpretation is an **update function** over information states:

$$s[Rt_1, \dots, t_n]] = \{i \in s \mid \langle i(t_1), \dots, i(t_n) \rangle \in i(R)\};$$

$$s[\neg \phi]] = \{i \in s \mid \neg \exists i' : i \leq i' \& i' \in s[\phi]]\};$$

$$s[\phi \land \psi]] = s[\phi]][\psi]];$$

$$s[[\Diamond \phi]] = \{i \in s \mid s[\phi]] \neq \emptyset\};$$

$$s[\exists x \phi]] = \bigcup_{d \in D} (s[x/d][\phi]), \text{ with}$$

$$s[x/d] = \{i[x/d] \mid i \in s\}$$

Support $s \models \phi$: *s* supports ϕ , $s \models \phi$, iff $\forall i \in s \exists i' \in s \llbracket \phi \rrbracket$: $i \leq i'$ (in *M*)

Entailment: ϕ entails ψ iff $s \llbracket \phi \rrbracket \models \psi$ (in all *M*)

Consistency: ϕ is consistent iff $s[\![\phi]\!] \neq \emptyset$ for some *s* (in some M)

Coherence: ϕ is coherent iff *s* supports ϕ for some $s \neq \emptyset$ (in some *M*)