Exercises Structures for Semantics

Here a selection of exercises related to the materials I used for the tutorial and assessment components of the course *Structures for Semantics* during the summer terms of 2021-2023 while I was teaching assistant for the course. The full course includes many more exercises and materials from earlier editions. Since I was not the sole contributor to these materials, I am not making them publicly available here. If you would like access to them, please reach out.

1 Indefinites

1.1 *The* and type-shifting rules

Consider the following GQT definition for *the*:

$$(the\ [n])(A) = \begin{cases} every(A) & \text{if } |A| = n \\ \text{undefined} & \text{otherwise} \end{cases}$$

(i) Assume that $|man| = 1$. Determine whether $the[1](man)$ is a filter, an ideal or an ultrafilter of the powerset lattice $\langle \wp(D), \subseteq \rangle$, based on the domain D. Provide proofs of your claims.

(ii) Assume that $|man| = 1$. Determine whether the following are equivalent or not. Motivate your answer.

  (a) $BE(the[1](man)) \equiv ident(lower(the[1](man)))$

  (b) $BE(the[1](man)) \equiv BE(lift(iota(man)))$

(iii) Consider now the set-theoretic interpretation of THE (man), where THE is Montague's translation of the definite article in English:

$$THE = \lambda P \lambda Q(\exists x(\forall y(P(y) \leftrightarrow y = x) \wedge Q(x)))$$

Does the following equation hold? Motivate your answer.

  (c) $THE(man) \equiv the[1](man)$

(iv) Assume $[\![W]\!] = \{a, b\} = woman$. Determine whether $the[2](woman)$ is a filter, an ideal or an ultrafilter of the powerset lattice $\langle \wp(D), \subseteq \rangle$, based on the domain D. No proofs needed. Consider now Landman's translation of *the women* using the σ operator: $\sigma x.{}^{\uparrow}W(x)$. Show that the following equation does not hold.

(d) $[\![\sigma x.{}^{\uparrow}W(x)]\!] \equiv the[2](woman)$

(v) Define type-shifting rules which can be applied to $the[2](woman)$ to verify the statement in (iv-d).

1.2 Indefinites and Team Semantics

Consider the following ambiguous sentence:
(A) Ali wants to marry a philosopher.

(i) Outline the ambiguity of (A). Provide translations of the two readings of (S) using Aloni & Degano (2022) dependence atoms (you should translate WANT in terms of a universal quantification over worlds).

(ii) Consider now the following variant of (A) involving a marked indefinite determiner IND_x triggering the activation of the variation atom $\text{var}(v, x)$. Which one of the two readings of (A) do Aloni & Degano (2022) predict for (B)?

(B) Ali wants to marry IND_x philosopher.

2 Generalized Quantifier Theory

2.1 Possesives

1. Find the GQT characterization of the determiners in (a) and (b);

   (a) *Every book*

   (b) *John's books*

2. Show that (a) satisfies ISOM, while (b) does not.

2.2 Connectedness/Convexity

(CON) A determiner Det is right *connected/convex* iff for all M with $A, B_2 \subseteq M$ and $B_1 \subseteq B \subseteq B_2$,

$$Det_M(A, B_1) \text{ and } Det_M(A, B_2) \text{ imply } Det_M(A, B)$$

(from van Benthem 1984)

For the following exercises, consider only determiners which can be represented in the Tree of Numbers (i.e., EXT, CONS and ISOM are satisfied)

(i) Give two examples of natural language determiners which are downward monotone on the right (i.e., $MON \downarrow$).

(ii) Give two examples of natural language determiners which are connected, but not monotone on any argument.

(iii) Represent the determiners you found in part (i) and (ii) in the Tree of Numbers. Which pattern do $MON \downarrow$ determiners exhibit? Which pattern do CON determiners exhibit?

3 Intensions

3.1 Ups and Downs

Assume the following type declarations.

IL **Declarations:**

Type	*Variables*	*Constants*
e	x	j
$\langle s, e \rangle$	r	-
$\langle e, t \rangle$	X	W
$\langle \langle s, e \rangle, t \rangle$	Q	C
$\langle s, \langle e, t \rangle \rangle$	P	-
$\langle s, t \rangle$	p	-

Determine if the following pair of expression are logically equivalent or not. No proofs needed: answering Equivalent/Non-Equivalent is sufficient. (/If not, construct a partial model in IL in which the two expression have different values.)

1.	j	${}^{\vee\wedge}j$
2.	r	${}^{\wedge\vee}r$
3.	$\lambda p\ \Box {}^{\vee}p({}^{\wedge}C({}^{\wedge}j)$	$\Box C({}^{\wedge}j)$
4.	$\lambda X\ \Box X(j)(\lambda x W(x)$	$\Box W(j)$
5.	$\lambda P\ \Box {}^{\vee}P(j)({}^{\wedge}\lambda x W(x))$	$\Box W(x)$
6.	$\lambda Q\ \Box Q({}^{\wedge}j)(\lambda r C(r))$	$\Box C({}^{\wedge}j)$

3.2 De re and de dicto

The sentence below is ambiguous between a *de re* and *de dicto* reading. (You can treat 'Miss Netherlands' as an individual constant.)

(1) John believes that Miss Netherlands is a dancer.

a. *De re*: John has a belief about a certain individual called 'Miss Netherlands' in the current world, the belief being that this individual is a dancer.

b. *De dicto*: John believes that whoever is named as 'Miss Netherlands' is a dancer.

Translate the two readings into IL and Ty2. Show using Theorem 6 from Gamut (p. 136) that the IL and Ty2 translations are equivalent.

4 Extensional Montague Grammar

4.1 Exceptive constructions

Extend the EMG fragment with exceptive constructions:

(2) Every student *but* John passed (the course).

(i) Provide an extension of EMG where *but* has category $T/(CN/CN)$. What are the problems of such analysis?

(ii) Provide now an extension of EMG which does not suffer from the problems you found before. Does your analysis overgenerate?

4.2 Pre-nominal adjectives in EMG

Extend the fragment of EMG presented in the EMG notes to account for 'pre-nominal' adjectives like *excellent* below:

(3) John is an excellent singer.

Treat *be* as a particular transitive verb with the following translation:

$$\text{BE: } \lambda X \lambda x\ X(\lambda y(x = y))$$

Consider the contrast below. How to account for this in EMG?

(4) a. John is an excellent singer.

b. $\Rightarrow$ John is a singer.

(5) a. John is a former singer.

b. $\not\Rightarrow$ John is a singer.

Definitions

EMG

$S2$: If $\alpha \in P_{(S/IV)=T}$ and $\beta \in P_{IV}$, then $F_1(\alpha, \beta) \in P_S$, where $F_1(\alpha, \beta) = \alpha\beta'$ (β' is β + inflection)

$T2$: If $\alpha \in P_T$ and $\beta \in P_{IV}$, and $\alpha \mapsto \alpha'$ and $\beta \mapsto \beta'$, then $F_1(\alpha, \beta) \mapsto \alpha'(\beta')$

$S'3$: If $\alpha \in P_{T/CN}$ and $\beta \in P_{CN}$, then $F_2(\alpha, \beta) \in P_T$, where $F_2(\alpha, \beta) = \alpha\beta$

$T'3$: If $\alpha \in P_{T/CN}$ and $\beta \in P_{CN}$, and $\alpha \mapsto \alpha'$ and $\beta \mapsto \beta'$, then $F_2(\alpha, \beta) \mapsto \alpha'(\beta')$

$S7$: If $\alpha \in P_{(IV/(S/IV))=TV}$ and $\beta \in P_T$, then $F_6(\alpha, \beta) \in P_{IV}$, where $F_6(\alpha, \beta) = \alpha\beta^*$ (β^* is β + accusative)

$T7$: If $\alpha \in P_{TV}$ and $\beta \in P_T$, and $\alpha \mapsto \alpha'$ and $\beta \mapsto \beta'$, then $F_6(\alpha, \beta) \mapsto \alpha'(\beta')$

$S8_n$: If $\alpha \in P_T$ and $\beta \in P_S$, then $F_7(\alpha, \beta) \in P_S$, where $F_7(\alpha, \beta) = \beta[he_n/\alpha]$

$T8_n$: If $\alpha \in P_T$ and $\beta \in P_S$, and $\alpha \mapsto \alpha'$ and $\beta \mapsto \beta'$, then $F_7(\alpha, \beta) \mapsto \alpha'(\lambda x_n \beta')$

$love \mapsto \lambda\mathcal{T}\lambda x(\mathcal{T}(\lambda y(love(y)(x))))$
$every \mapsto \lambda P\lambda Q(\forall x(P(x) \rightarrow Q(x)))$
$a = \lambda P\lambda Q(\exists x(P(x) \wedge Q(x)))$

IL Semantic Clauses

If α is a constant, $[\![\alpha]\!]_{M,w,g} = I(\alpha)(w)$

If α is a variable, $[\![\alpha]\!]_{M,w,g} = g(\alpha)$

If α is an expression of type $\langle a, b\rangle$ and β an expression of type a, $[\![\alpha(\beta)]\!]_{M,w,g} = [\![\alpha]\!]_{M,w,g}([\![\beta]\!]_{M,w,g})$

If α is an expression of type a and z variable of type b, $[\![\lambda z\alpha]\!]_{M,w,g}$ is that function $h \in D_{\langle b,a\rangle}$ s.t. for all $d \in D_b$: $h(d) = [\![\alpha]\!]_{M,w,g[z/d]}$

$[\![\Box\phi]\!]_{M,w,g} = 1$ iff $\forall w' \in W : [\![\phi]\!]_{M,w',g} = 1$

If α is an expression of type a, then $[\![{}^\wedge\alpha]\!]_{M,w,g}$ is that function $h \in D_{\langle s,a\rangle}$ such that for all $w' \in W$: $h(w') = [\![\alpha]\!]_{M,w',g}$

If α is an expression of type $\langle s, a\rangle$, then $[\![{}^\vee\alpha]\!]_{M,w,g} = [\![\alpha]\!]_{M,w,g}(w)$

IL - Ty2 translation

(i) $\sigma(c_\tau) = c_{\langle s,\tau\rangle}(v)$
$\sigma(v_\tau) = v_\tau$
(ii) $\sigma(\alpha(\beta)) = (\sigma(\alpha)(\sigma(\beta)))$
(iii) $\sigma(\neg\phi) = \neg\sigma(\phi)$
(iv) $\sigma(\phi \wedge \psi) = \sigma(\phi) \wedge \sigma(\phi)$ [likewise for $\vee, \rightarrow, \leftrightarrow$]
(v) $\sigma(\forall x(\phi)) = \forall x(\sigma(\phi))$ [likewise for $\exists x(\phi)$]
(vi) $\sigma(\alpha = \beta) = \sigma(\alpha) = \sigma(\beta)$
(vii) $\sigma(\lambda x(\alpha)) = \lambda x(\sigma(\alpha))$
(viii) $\sigma(\Box\phi) = \forall v(\sigma(\phi))$
(ix) $\sigma(\Diamond\phi) = \exists v(\sigma(\phi))$
(x) $\sigma({}^\wedge\alpha) = \lambda v(\sigma(\alpha))$
(xi) $\sigma({}^\vee\alpha) = (\sigma(\alpha(v)))$

Theorem 6: $[\![\sigma(\alpha)]\!]_{M2,g[v/w]} = [\![\alpha]\!]_{M,w,g}$

Plurals

The language

1. The standard first order operations $\neg, \wedge, \vee, \exists$ and abstraction λ.
2. Individual constants and individual variables.
3. Two term creating operations: + for term conjunction and σ for definites.
4. A special relational constant $\leq$.
5. A set **P** of one place predicates. This set is sorted into three different sets:
   (a) IND: the set of individual level predicates
   (b) COL: the set of collective predicates
   (c) MIX: the set of mixed predicates
6. A special predicate $AT \in IND$
7. Three predicate operations: $^\uparrow, ^\downarrow, ^D$

Models

A model for LP is a triple $\langle\langle A, V\rangle, *, I\rangle$ where:

1. $\langle A, \vee\rangle$ is a free i-join (=complete) semilattice generated by a set of atoms AT. $PL = A \backslash AT$
2. $* \notin A$ (undefined element to deal with non-referring terms)
3. I is an interpretation function, such that
   - If $c \in CON$, then $I(c) \in A \cup \{*\}$
   - If $P \in IND$, then $I(P) \subseteq AT$
   - If $P \in COL$, then $I(P) \subseteq PL$
   - If $P \in MIX$, then $I(P) \subseteq A$

Semantics

Terms:

$[\![t_1 + t_2]\!] = [\![t_1]\!] \cup [\![t_2]\!]$, if both $[\![t_1]\!], [\![t_2]\!] \in A$; * otherwise

$[\![\sigma x.P(x)]\!] = \bigvee[\![P]\!]$, if $\bigvee[\![P]\!] \in [\![P]\!]$; * otherwise

Predicates:

$[\![AT]\!] = AT$

$[\![{}^\uparrow P]\!] = [[\![P]\!]]$, the complete sub join-semilattice of A generated by $[\![P]\!]$ [contains all the individual joins of members of $[\![P]\!]$]

$[\![{}^\downarrow P]\!] = \{d \in AT : d \in [\![P]\!]\}$

Formulas:

$[\![P(t)]\!] = 1$ iff $[\![t]\!] \in [\![P]\!]$, 0 otherwise

$[\![t \leq t']\!] = 1$ iff $[\![t]\!] \leq [\![t']\!]$, 0 otherwise

Filter, Ideal, Ultrafilter

Let $\langle A, \leq\rangle$ be a lattice. A subset $X \subseteq A$ is:

- *upward closed* if $a \in X$ and $a \leq b$ implies $b \in X$;
- *downward closed* if $b \in X$ and $a \leq b$ implies $a \in X$;
- *a filter* if it is (1) non-empty, (2) upward closed, (3) closed under binary meet: if $a, b \in X$ then $a \wedge b \in X$

- *an ideal* if it is: (1) non-empty, (2) downward closed, (3) closed under binary join: if $a, b \in X$ then $a \vee b \in X$

Let $\langle A, \leq \rangle$ be a Boolean lattice. $X \subseteq A$ is an *ultrafilter* if:

1. it is a filter;
2. for any $a \in A$, exactly one of a and its complement is in X

Let $\langle A, \leq \rangle$ be a Boolean lattice. A (ultra)filter $F \subseteq A$ is *principal* if there exists a set S, with $S \neq \emptyset$ and $S \subseteq A$, s.t. $F = \{B : S \subseteq B\}$. We call S the *generator* of the principal (ultra)filter F.

Generalized Quantifiers

ISOM, EXT and CONS

(ISOM) A determiner D is topic-neutral iff for any M, M' and any $A, B \subseteq M$, $A', B' \subseteq M'$:
If $(M, A, B) \cong (M', A', B')$, then $D_M(A, B) \leftrightarrow D'_M(A', B')$

(EXT) A determiner D satisfies extension iff for any M and any $A, B \subseteq M$:
If $M \subseteq M'$, then $D_M(A, B) \Leftrightarrow D_{M'}(A, B)$

(CONS) A determiner D is conservative iff for any M and any $A, B \subseteq M$:
$D_M(A, B) \Leftrightarrow D_M(A, A \cap B)$ *Monotonicity (fixing a model M)*

MON↑: A determiner D is **right monotone increasing** iff
$B \subseteq B'$ and $D(A)(B)$ then $D(A)(B')$

MON↓. A determiner D is **right monotone decreasing** iff
$B \subseteq B'$ and $D(A)(B')$ then $D(A)(B)$

↑MON. A determiner D is **left monotone increasing** iff
$A \subseteq A'$ and $D(A)(B)$ then $D(A')(B)$

↓MON. A determiner D is **left monotone decreasing** iff
$A \subseteq A'$ and $D(A')(B)$ then $D(A)(B)$

Tree of Numbers

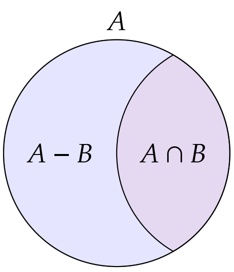
(0, 0)

(1, 0) (0, 1)

(2, 0) (1, 1) (0, 2)

(3, 0) (2, 1) (1, 2) (0, 3)

... ...



Each position in the tree corresponds to pairs $(|A - B|, |A \cap B|)$

Each row in the tree corresponds to a different cardinality of A:
Row$_0$: $card(A) = 0$,
Row$_1$: $card(A) = 1, \ldots$

+ indicates that the quantifier is true in that situation.
− indicates that the quantifier is false in that situation.
Examples:

Every	Some
+	-
- +	- +
- - +	- + +
- - - +	- + + +

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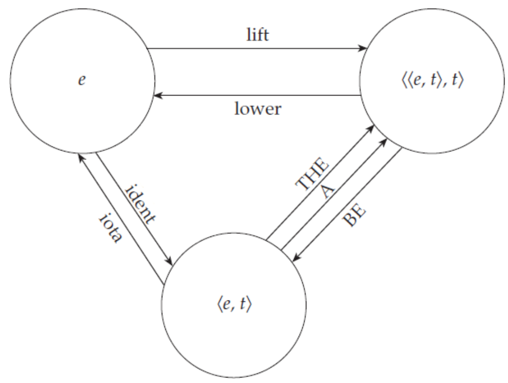
Type-Shifting



$BE = \lambda T_{\langle\langle e,t\rangle,t\rangle} \lambda x_e (T(\lambda y_e (y = x)))$

$THE = \lambda P_{\langle e,t\rangle} \lambda Q_{\langle e,t\rangle} (\exists x (\forall y (P(y) \leftrightarrow y = x) \wedge Q(x)))$

$A = \lambda P_{\langle e,t\rangle} \lambda Q_{\langle e,t\rangle} (\exists x (P(x) \wedge Q(x)))$

lift	$e \mapsto \langle\langle e,t\rangle,t\rangle$	$j \mapsto \lambda P P(j)$
lower	$\langle\langle e,t\rangle,t\rangle \mapsto e$	$lower(lift(j)) = j$

(lower maps a principal ultrafilter to the unique element in its generator)

ident	$e \mapsto \langle e,t\rangle$	$j \mapsto \lambda x (x = j)$
iota	$\langle e,t\rangle \mapsto e$	$P \mapsto \iota x P(x)$

(iota maps a property to the unique individual satisfying that property)

Team Semantics

$M, T \models P(x_1, \ldots, x_n)$	$\Leftrightarrow$	$\forall j \in T : \langle j(x_1), \ldots, j(x_n)\rangle \in I(P^n)$
$M, T \models \phi \wedge \psi$	$\Leftrightarrow$	$M, T \models \phi$ and $M, T \models \psi$
$M, T \models \phi \vee \psi$	$\Leftrightarrow$	$T = T_1 \cup T_2$ for two teams T_1 and T_2 s.t. $M, T_1 \models \phi$ and $M, T_2 \models \psi$
$M, T \models \forall y \phi$	$\Leftrightarrow$	$M, T[y] \models \phi$, where $T[y] = \{i[d/y] : i \in T$ and $d \in D\}$
$M, T \models \exists_{\text{strict}} y \phi$	$\Leftrightarrow$	there is a function $h : T \rightarrow D$ s.t. $M, T[h/y] \models \phi$, where $T[h/y] = \{i[h(i)/y] : i \in T\}$
$M, T \models \exists_{\text{lax}} y \phi$	$\Leftrightarrow$	there is a function $f : T \rightarrow \wp(D) \backslash \{\varnothing\}$ s.t. $M, T[f/y] \models \phi$, where $T[f/y] = \{i[d/y] : i \in T$ and $d \in f(i)\}$
$M, T \models dep(\vec{x}, y)$	$\Leftrightarrow$	for all $i, j \in T : i(\vec{x}) = j(\vec{x}) \Rightarrow i(y) = j(y)$
$M, T \models var(\vec{x}, y)$	$\Leftrightarrow$	there is $i, j \in T : i(\vec{x}) = j(\vec{x})$ & $i(y) \neq j(y)$